



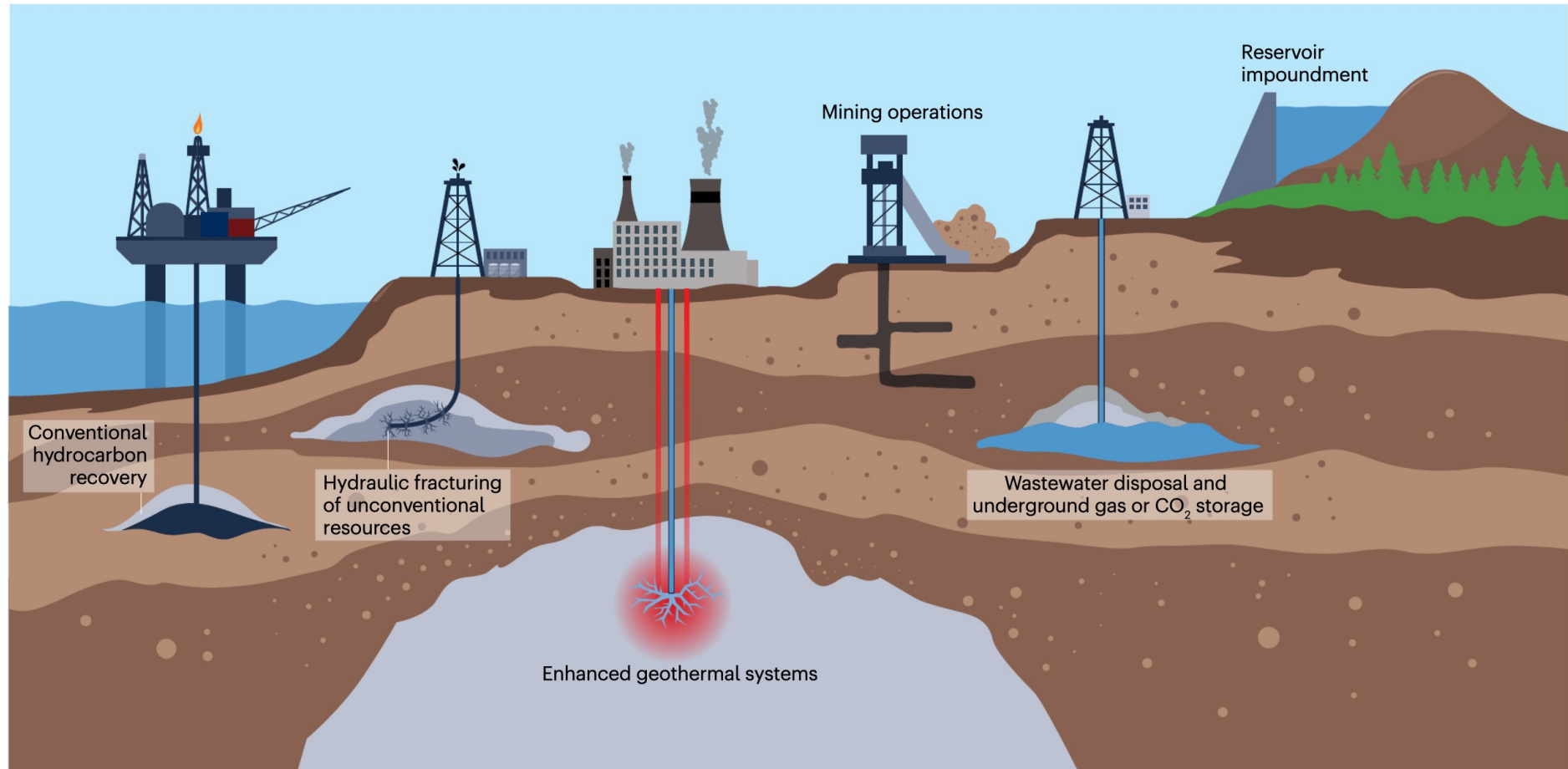
# The Physical Processes of Maximum Magnitudes of Induced Earthquakes

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# Industrial activities that can cause induced seismicity

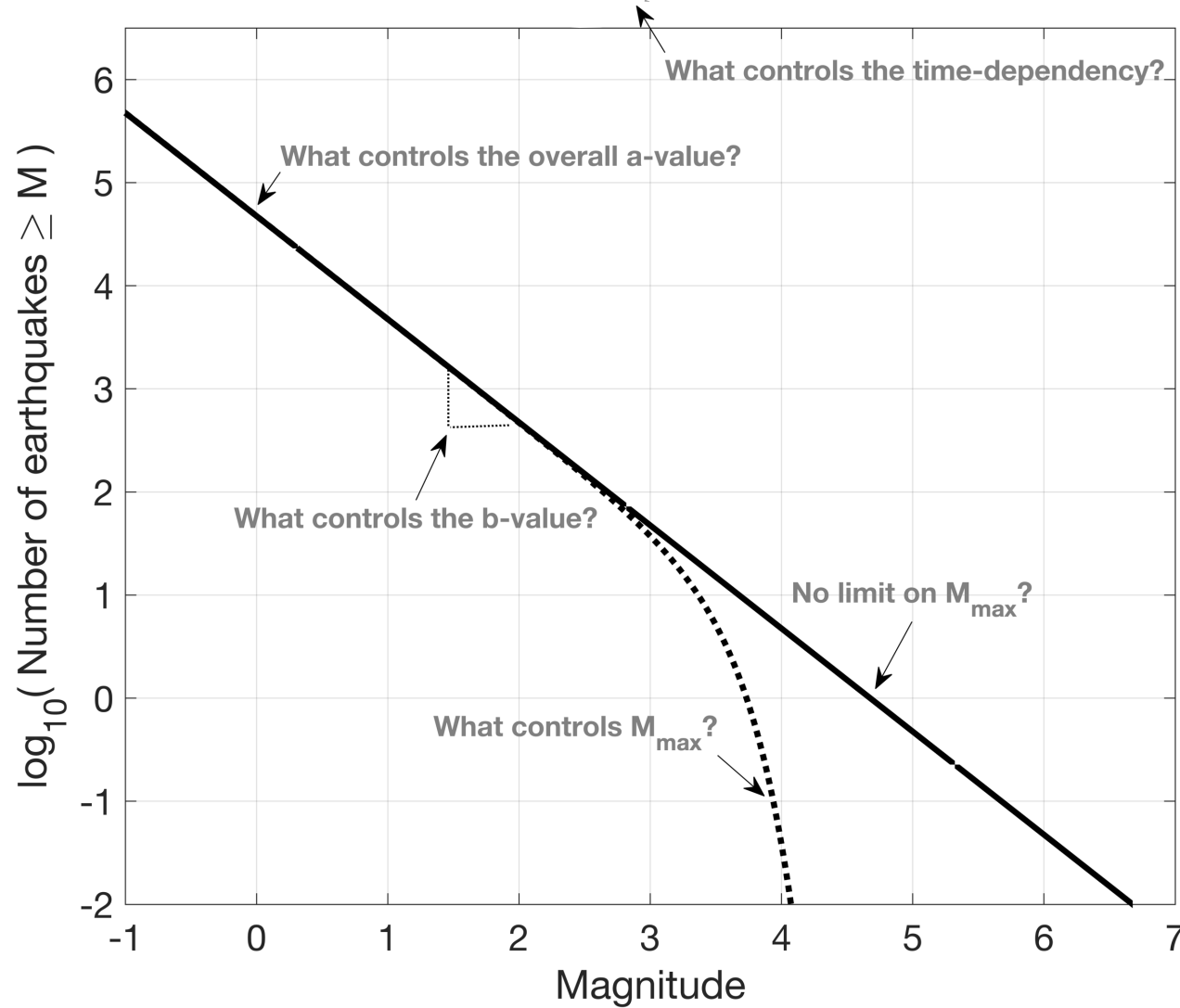


**Fig. 1 | Industrial activities that can cause induced seismicity.** Induced earthquakes can occur during conventional hydrocarbon recovery, hydraulic fracturing of unconventional resources, enhanced geothermal systems, mining

operations, wastewater disposal, underground gas or CO<sub>2</sub> storage operations and reservoir impoundment.

*Moein, Langenbruch, et al., 2023, Nature Reviews Earth & Environment*

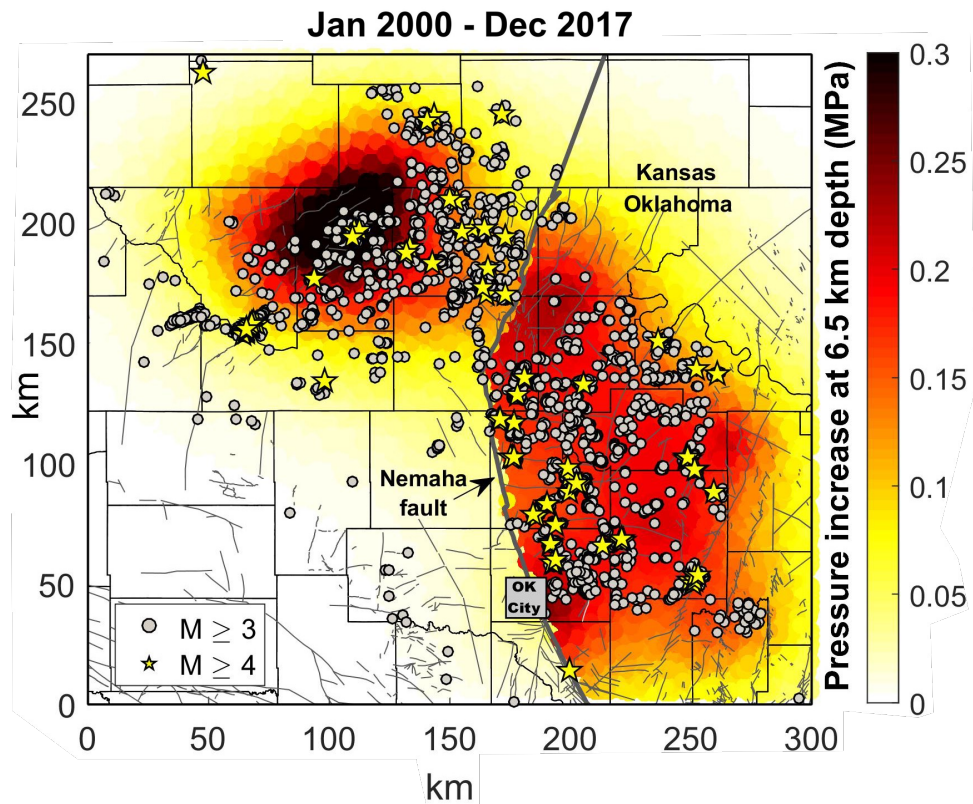
$$\log_{10} N_{\geq M}(t) = a(t) - b M$$



Challenges in  
forecasting the  
recurrence rates of  
induced earthquakes?

Water disposal

M=5.8, Pawnee, Oklahoma, USA, 2016



Langenbruch et. al, 2018, Nature Comm.



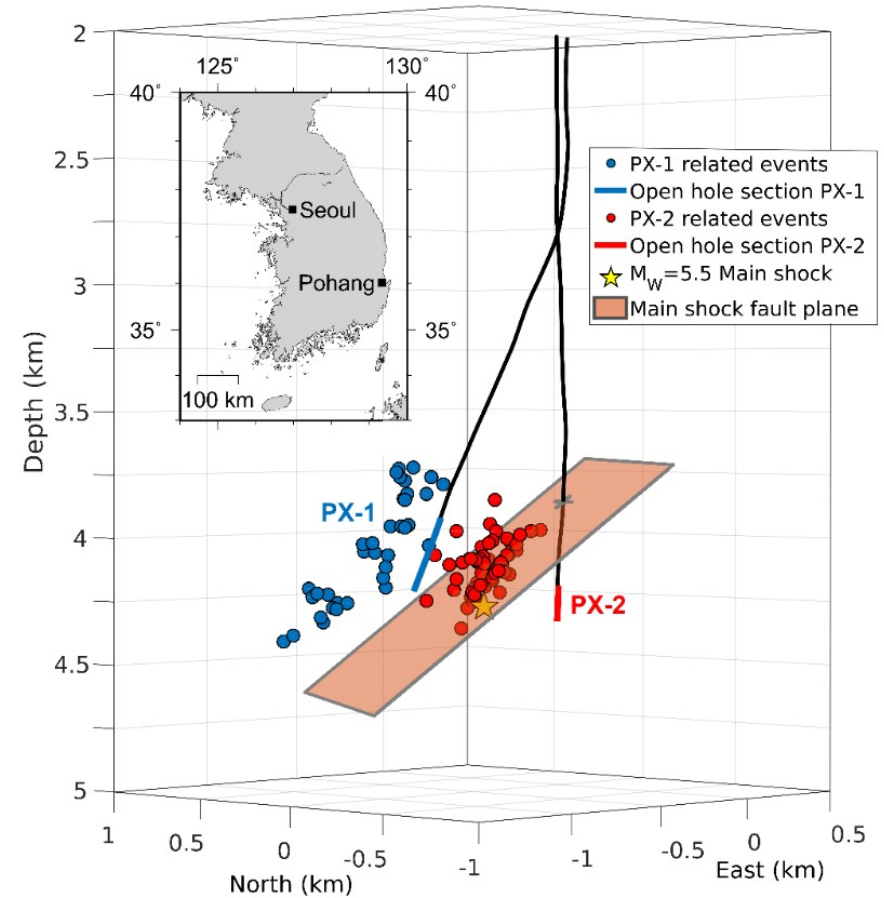
<https://www.nbcnews.com/news/us-news/state-emergency-declared-oklahoma-after-magnitude-5-6-earthquake-n642676>  
<https://arstechnica.com/science/2016/11/town-files-lawsuit-after-largest-earthquake-in-oklahoma-history/>



# M=5.5, Pohang, South Korea, 2017



## Enhanced Geothermal System (EGS)



Langenbruch et. al, 2020, GRL

<https://www.independent.co.uk/news/world/asia/south-korea-earthquake-latest-pohang-quake-second-strongest-record-5-4-magnitude-a8057946.html>  
<http://koreabizwire.com/life-on-hold-for-students-after-pohang-earthquake/101585>

M=6.3, Koyna, India, 1967

Reservoir impoundment



A view of a Koyna Dam in Satara, Maharashtra on Tuesday. Water level of the dam is decreasing due to hot summer. PTI Photo (PTI5\_2\_2017\_000151A)



PHILOSOPHICAL  
TRANSACTIONS A

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Research



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# Are maximum magnitudes of induced earthquakes controlled by pressure diffusion?

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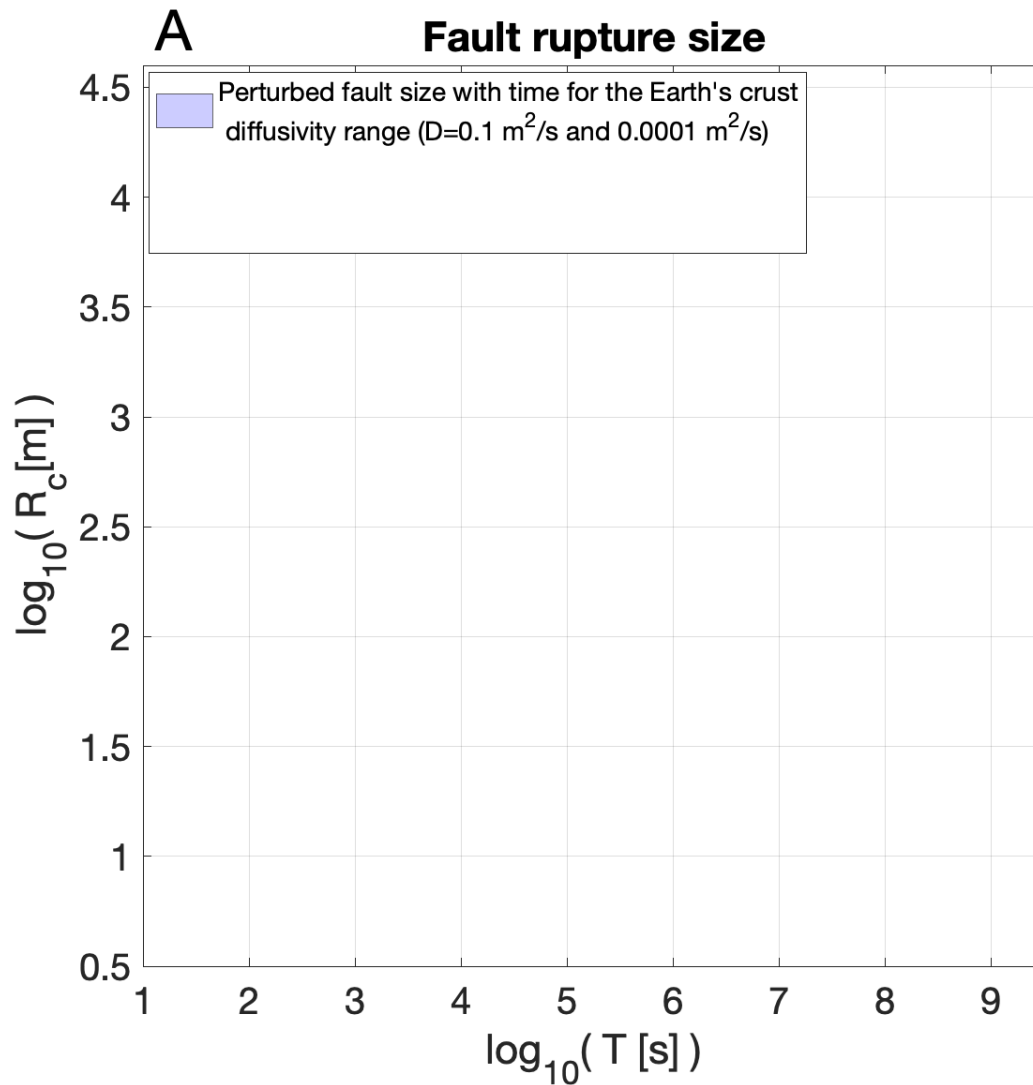
Cornelius Langenbruch, Mohammad J. A. Moein  
and Serge A. Shapiro

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## Pressure diffusion controlled growth of perturbed fault size (rock volume) with time?

Triggering front, *Shapiro et al., 1997, 2002*

$$R_c = \sqrt{4 \pi D T}$$

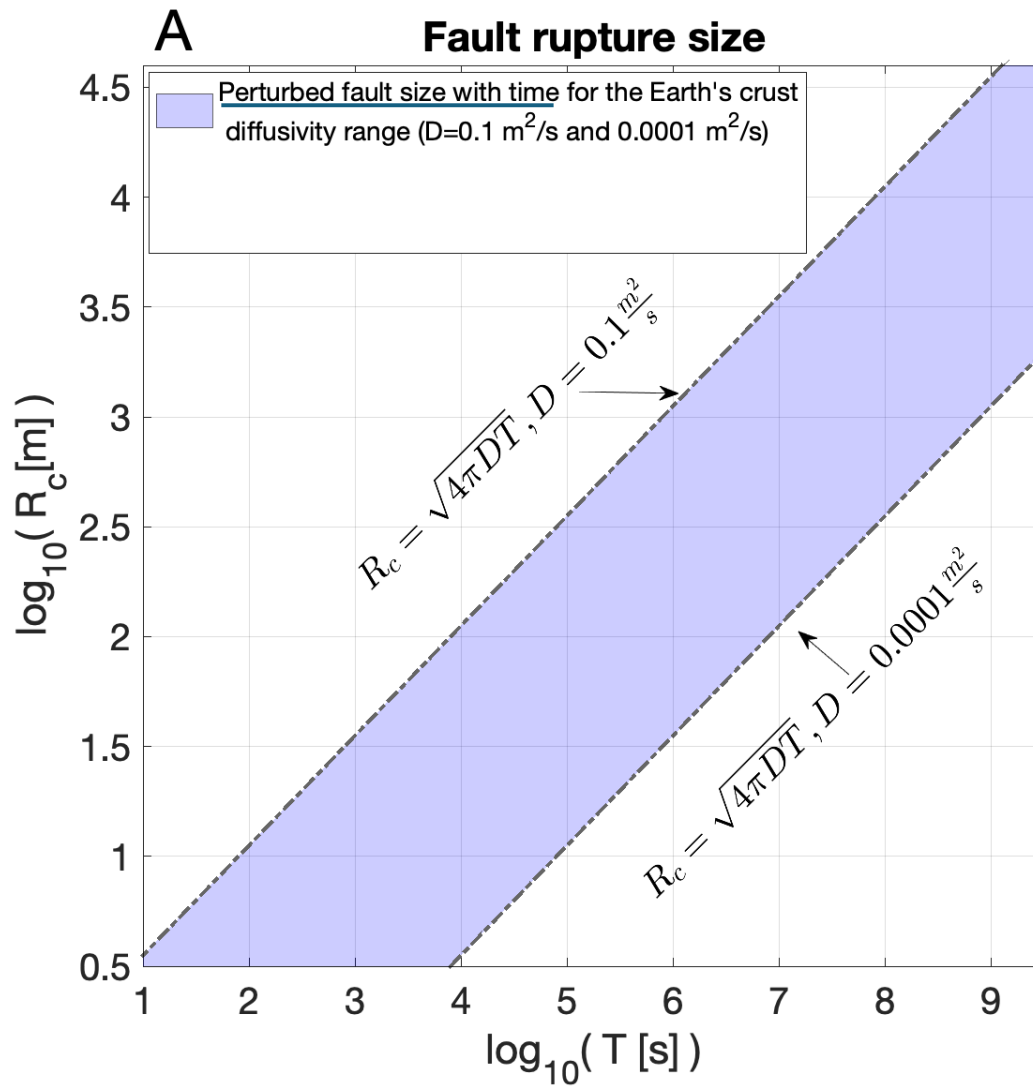
$$2 \log_{10} R_c = \log_{10} T + \log_{10} D + \log_{10} 4\pi.$$

### Two conditions:

- Pressure diffusion occurs predominantly along pre-existing critically stressed faults  
(*e.g. Townsend and Zoback, 2000*)
- The rupture size of induced earthquakes is in the order of the pressure perturbed part of a fault at occurrence time of an earthquake.

(*runaway ruptures and multi-physical processes are not excluded*)





## Pressure diffusion controlled growth of perturbed fault size (rock volume) with time?

Triggering front, *Shapiro et al., 1997, 2002*

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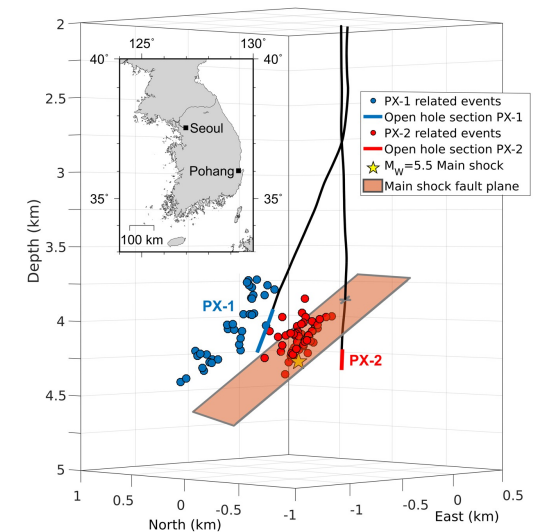
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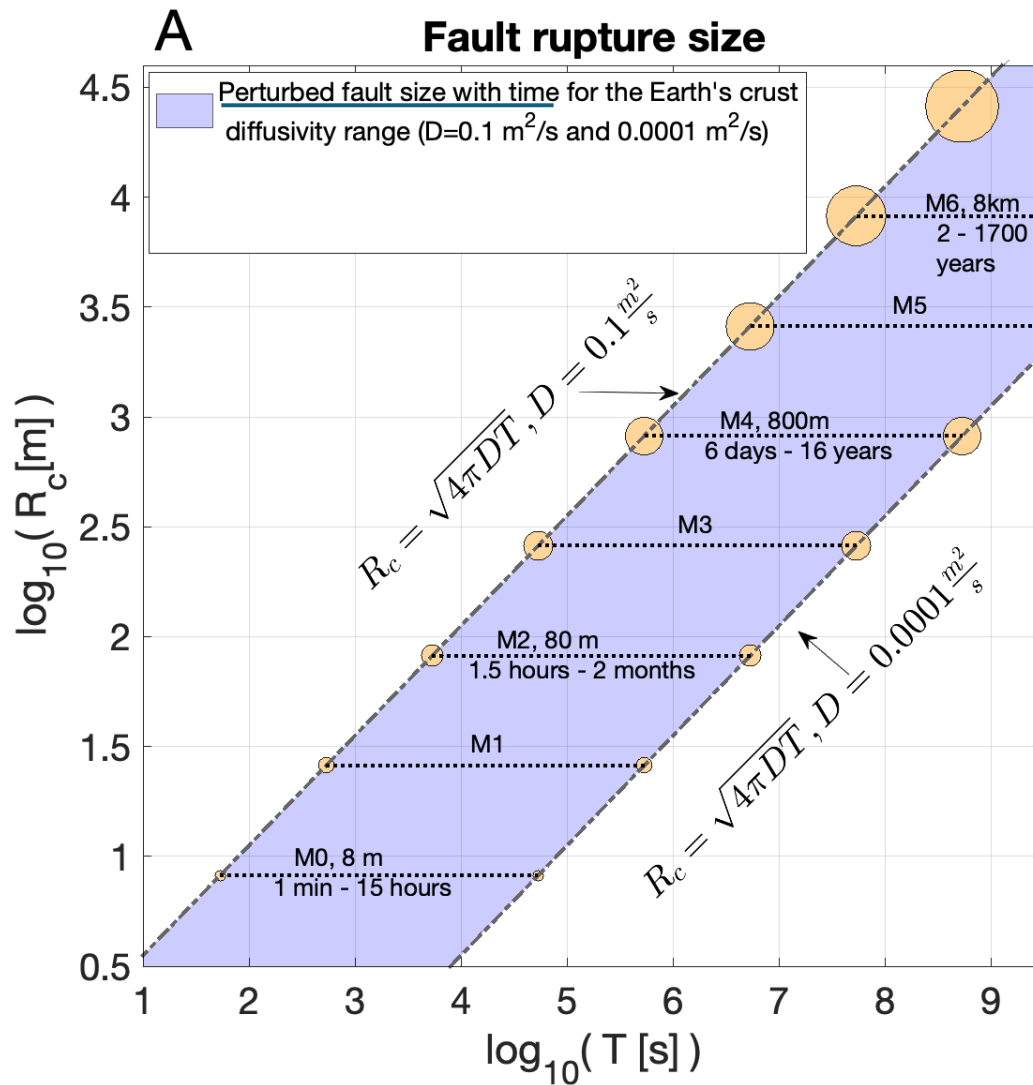
Brittle Earth's crust diffusivity range:

*Townend and Zoback, 2000*

$$D = 10^{-4} \text{ to } 10^{-1} \frac{\text{m}^2}{\text{s}}$$

Pressure diffusion occurs predominantly along pre-existing critically stressed faults.





## Pressure diffusion controlled growth of perturbed fault size (rock volume) with time?

$R_c$  can be considered as the pressure perturbed fault size!

What is the growth of the maximum magnitude over time if ruptures are in the order of the pressure perturbed part of a fault  $R_c$  at time  $T$ ?

Kanamori, 1977, Hanks and Kanamori, 1979, Madariaga, 1979, Lay Wallace, 1995:

$$M_W = \frac{2}{3} (\log_{10} M_0 - 9.1), \quad M_0 = \frac{16}{7} R_c^3 \Delta \sigma$$

Radius ( $R_c$ ) of a circular rupture, seismic moment ( $M_0$ ), moment magnitude ( $M_W$ ) and stress drop ( $\Delta \sigma$ )

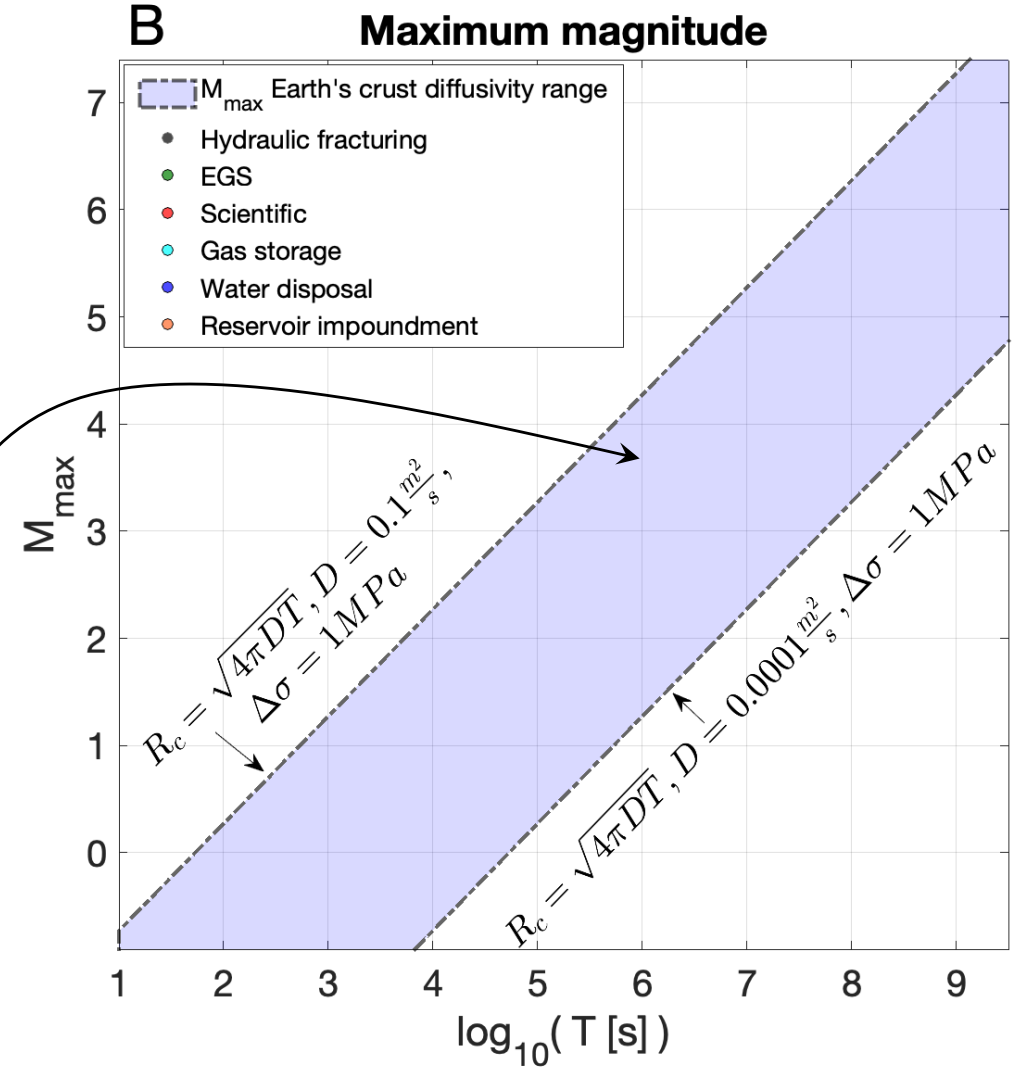
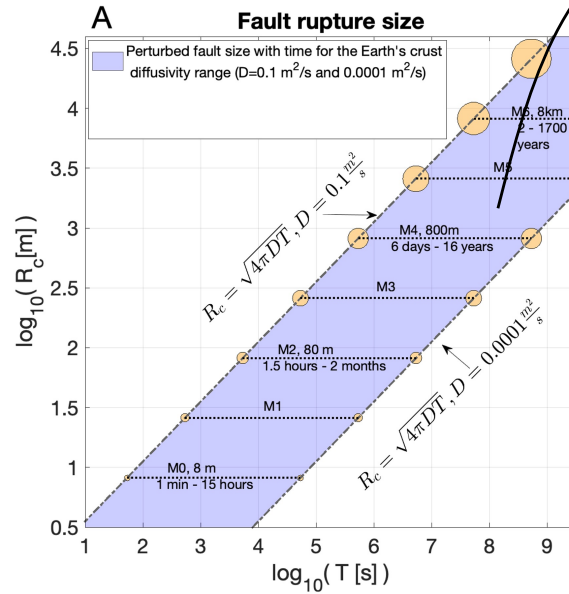
$$M_W = \frac{2}{3} (\log_{10} M_0 - 9.1), \quad M_0 = \frac{16}{7} R_c^3 \Delta\sigma$$

$$R_c = \sqrt{4 \pi D T}$$

$$M_W^{max} = \log_{10} T + \Theta_{D\sigma}$$

$$\Theta_{D\sigma} = \log_{10} D + \frac{2}{3} \log_{10} \Delta\sigma - 4.7281$$

The **seismic nucleation constant**  $\Theta_{D\sigma}$  controls the characteristic time to reach a subsurface state allowing a given magnitude earthquake to occur



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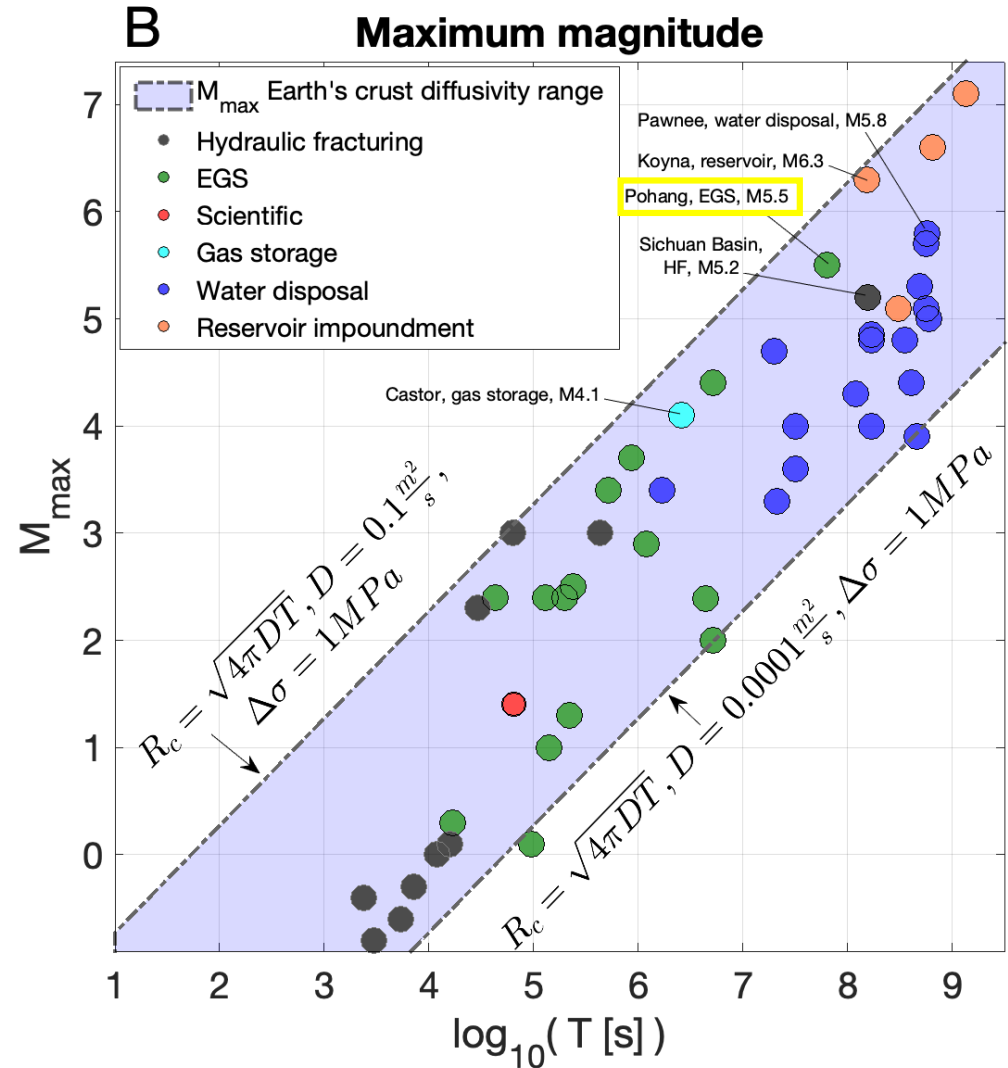
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The **seismic nucleation constant**  $\Theta_{D\sigma}$  controls the characteristic time to reach a subsurface state allowing a given magnitude earthquake to occur

$\Theta_{D\sigma}$ :  
Magnitude of  
nucleation  
time 1 second

Global compilation of maximum induced earthquake magnitudes  
hydraulic fracturing, geothermal reservoir stimulation, water disposal, gas storage and reservoir impoundment



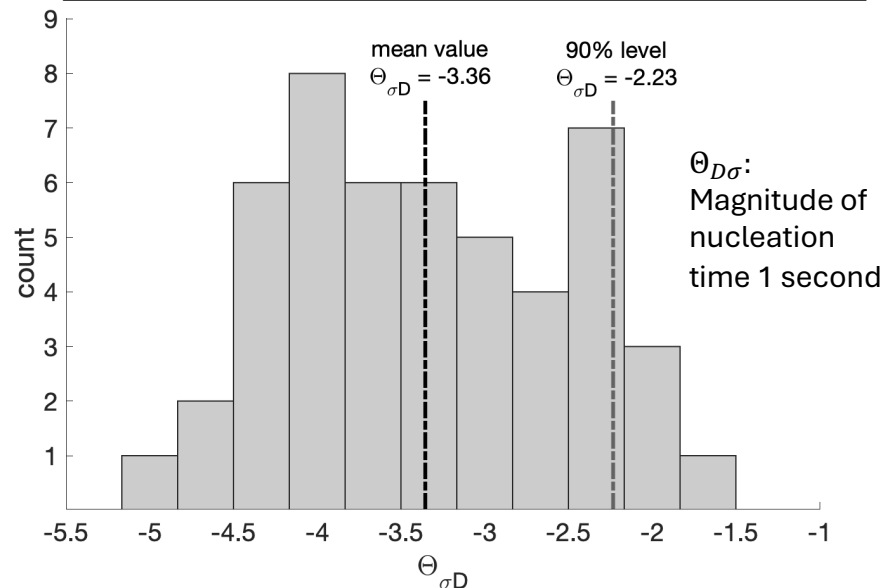


# How to calibrate a first order seismic hazard model?

Can we estimate the expected maximum magnitude prior to the start of a geo energy project?

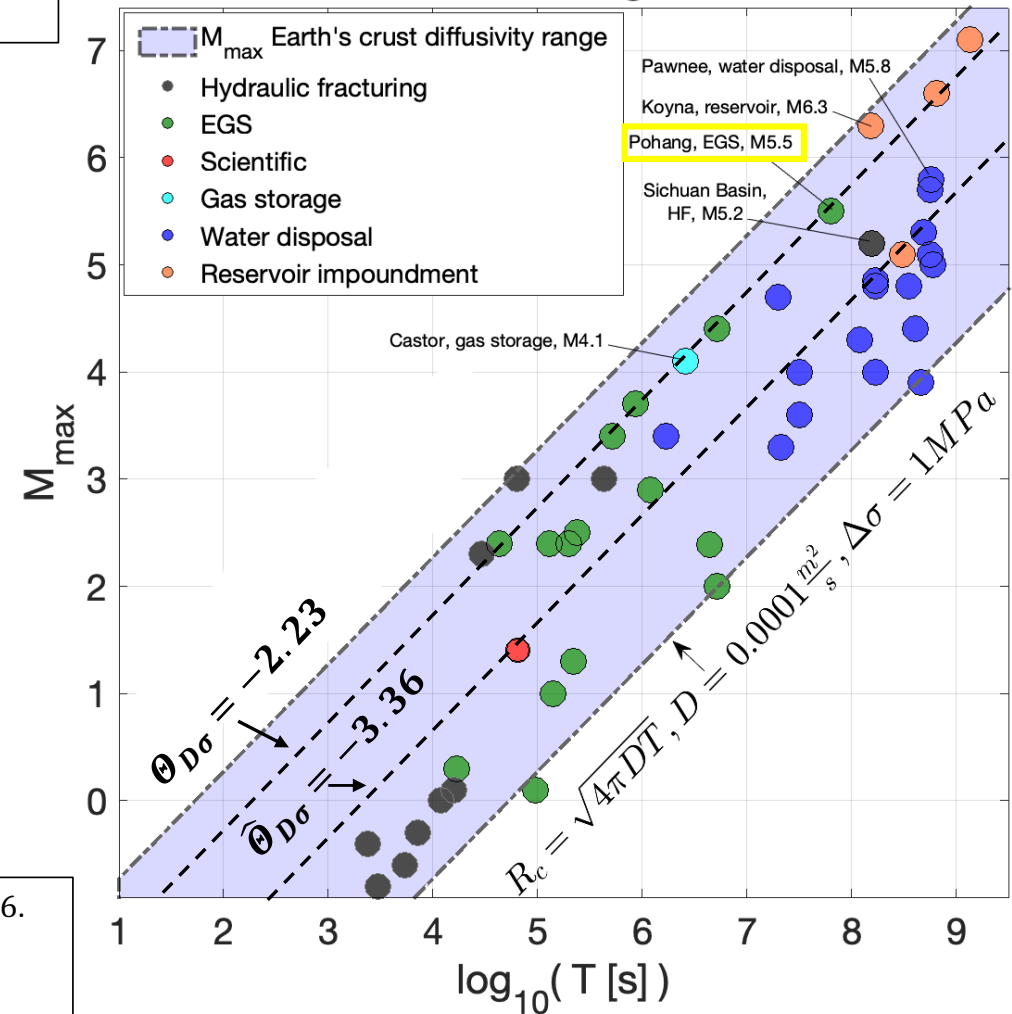
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The maximum expected induced magnitude scales according to  $\hat{\Theta}_{D\sigma} = -3.36$ .  
The upper bound (90% level) of  $M_{max}$  scales according to  $\Theta_{D\sigma} = -2.23$ .  
These quantities can be used in seismic hazard studies pre-operation!

## B Maximum magnitude

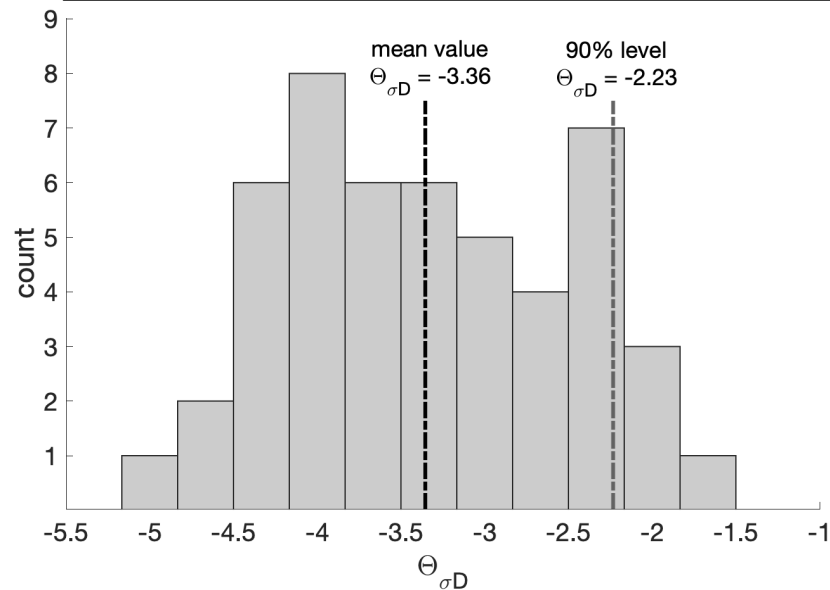


# How to calibrate a first order seismic hazard model?

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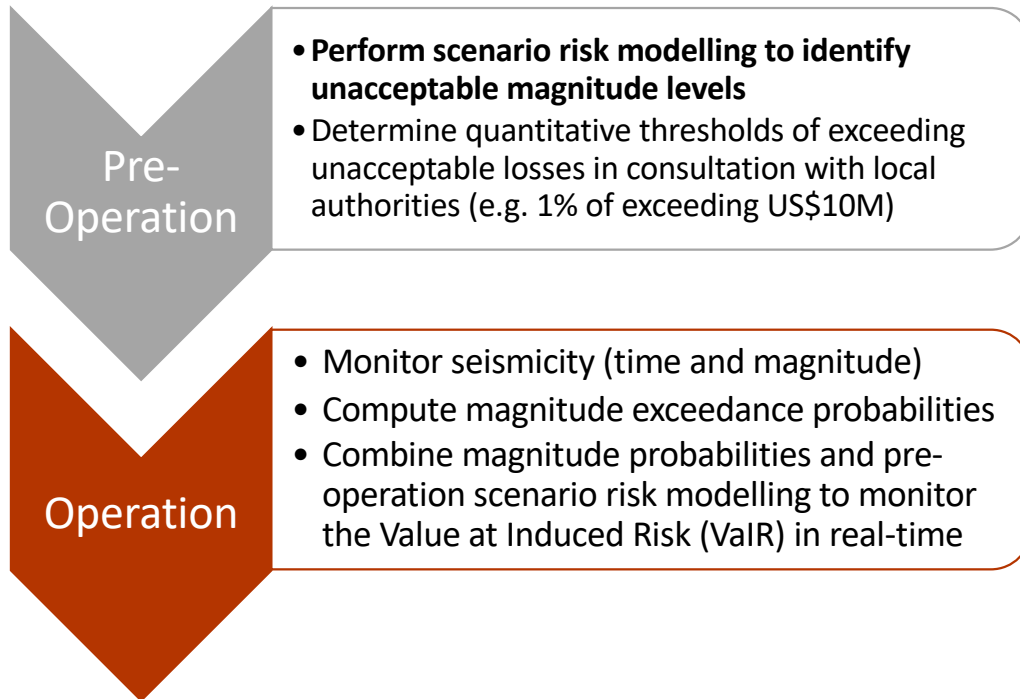
Our calibrated maximum magnitude model can be used pre-operation!

Nucleation time	M <sub>max</sub> upper bound (90%)	M <sub>max</sub> expected
1 hour	1.3	0.2
1 day	2.7	1.6
1 week	3.6	2.4
1 month	4.2	3.1
1 year	5.3	4.1
10 years	6.3	5.1
100 years	7.3	6.1

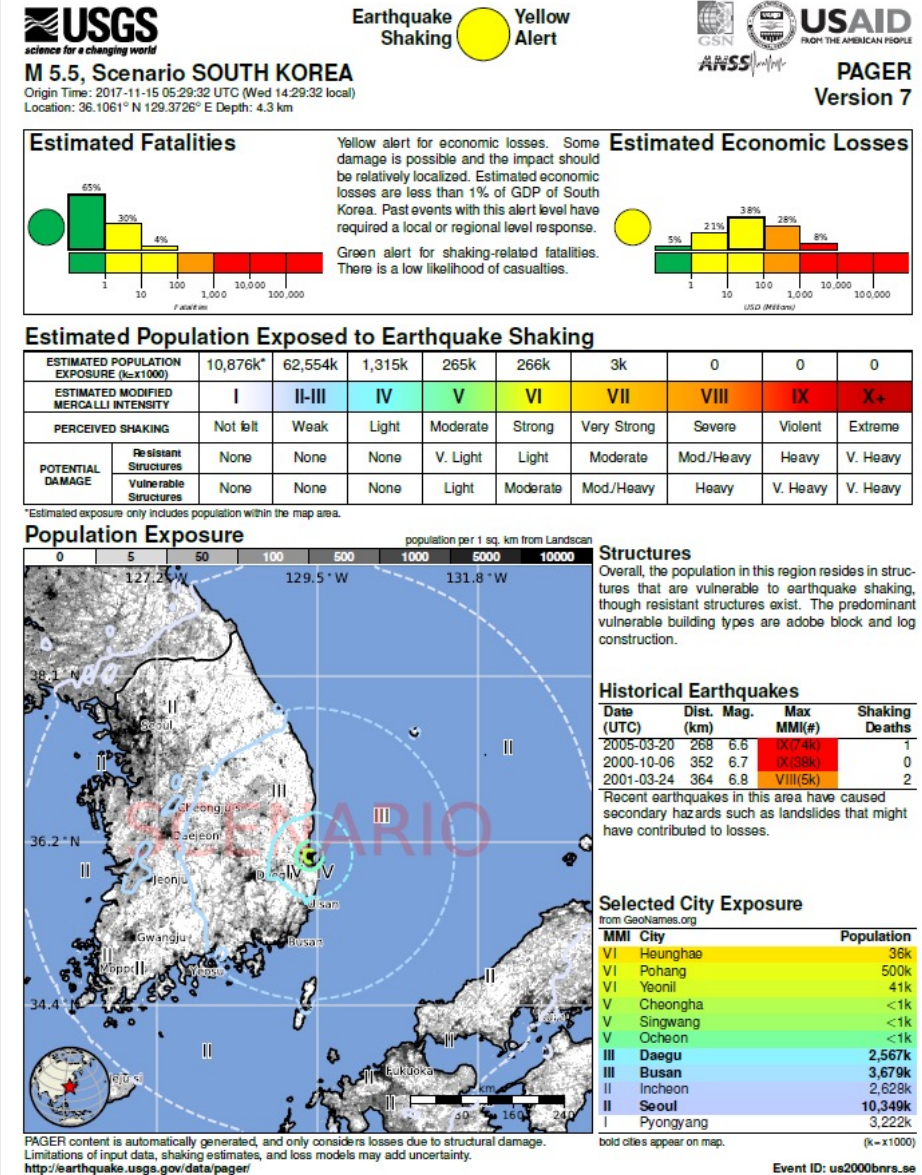
# Value at Induced Seismic Risk (VaIR)

A risk management approach based on low-probability high-impact events!

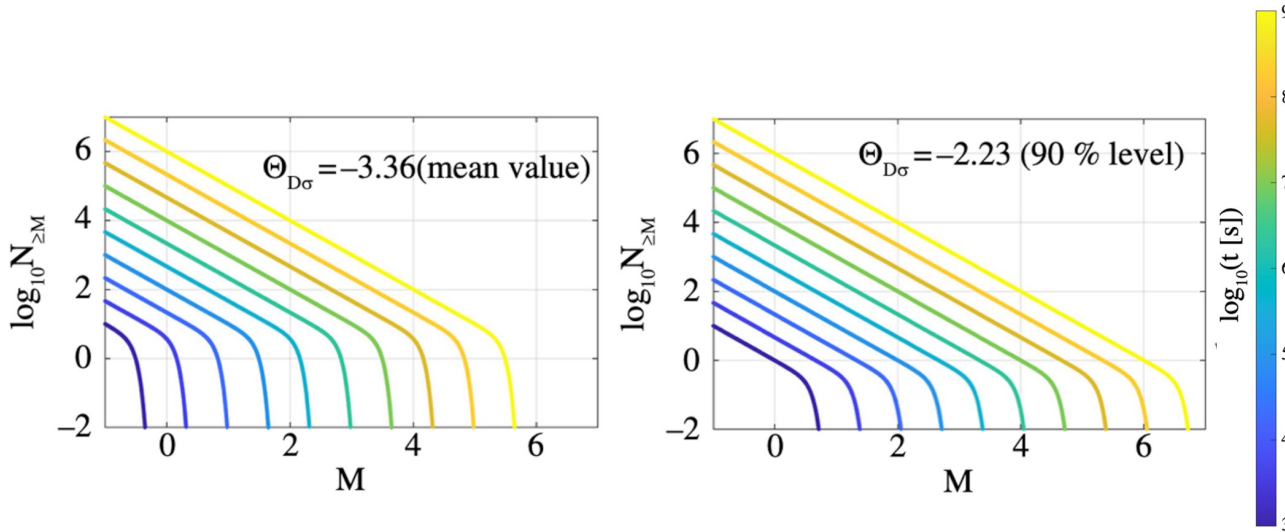
Langenbruch, Ellsworth, Woo and Wald, 2020, GRL



(PAGER) Prompt Assessment of Global Earthquakes for Response



# Physics-based truncation of the Gutenberg-Richter law and the Seismogenic Index



Gutenberg-Richter law:

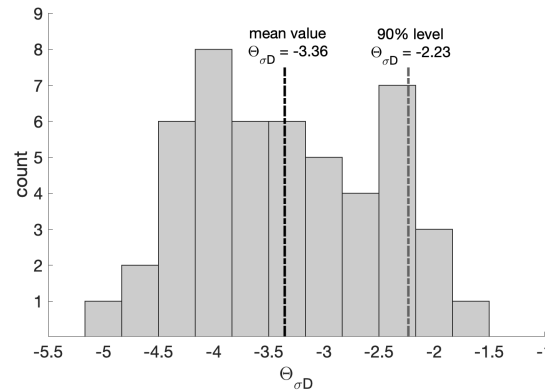
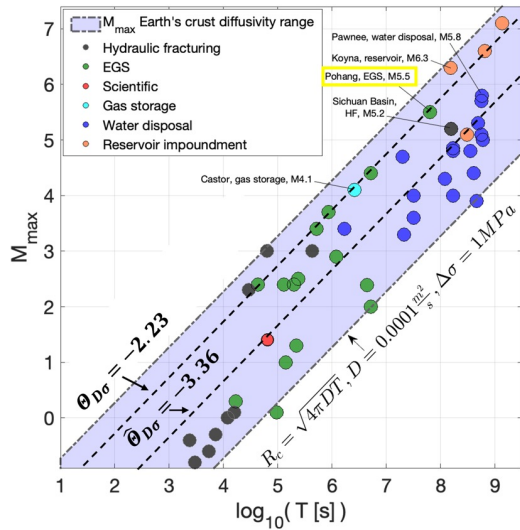
$$\log_{10}(N_{\geq M}) = a - bM$$

Exponential taper:

$$P(M) = \beta \left( e^{-\beta(M - M_c)} - \gamma e^{\delta(M - M_m)} \right), M > M_c,$$

*e.g. Kagan, 2010, Eaton et al., 2021*

$$\beta = \log(10)b,$$

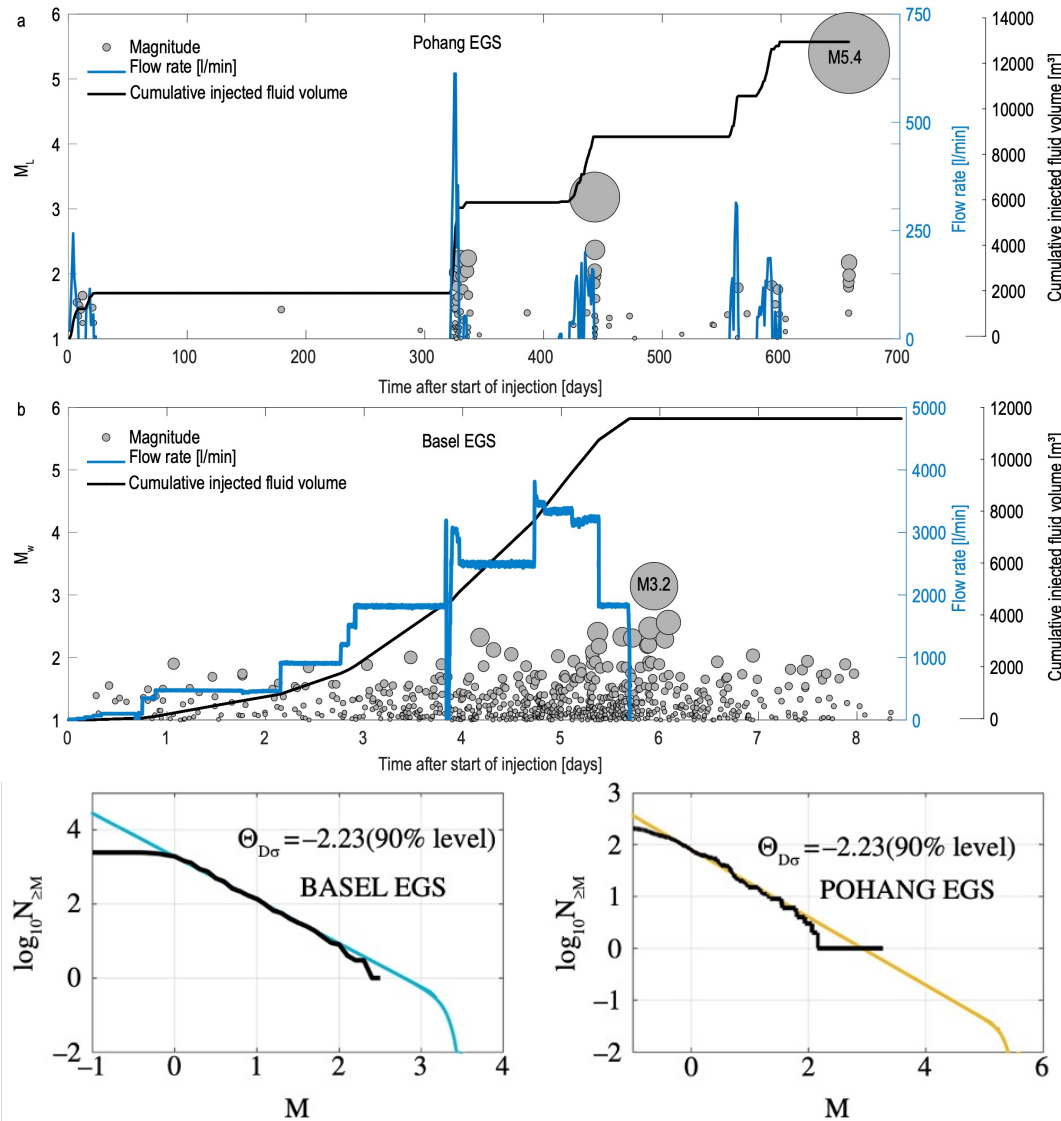


Corner magnitude  $M_m$  can be replaced by:

$$M_W^{max} = \log_{10}T + \Theta_{D\sigma}$$



# Physics-based truncation of the Gutenberg-Richter law and the Seismogenic Index



Gutenberg-Richter law:

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Corner magnitude  $M_m$  can be replaced by:

$$M_W^{max} = \log_{10} T + \Theta_{D\sigma}$$

**To forecast the complete magnitude frequency distribution we must combine the  $M_{max}$  model with a model describing the earthquake productivity (a-value)!**

# The Seismogenic Index

Shapiro, Dinske, Langenbruch and Wenzel, 2010

Injection  
Volume-based:

## Gutenberg-Richter law for fluid injection-induced seismicity

$$\log_{10} [N_{\geq M}(t)] = \underbrace{\log_{10} [V_I(t)] + \Sigma}_{\text{a-value of the classical GR relation}} - bM$$

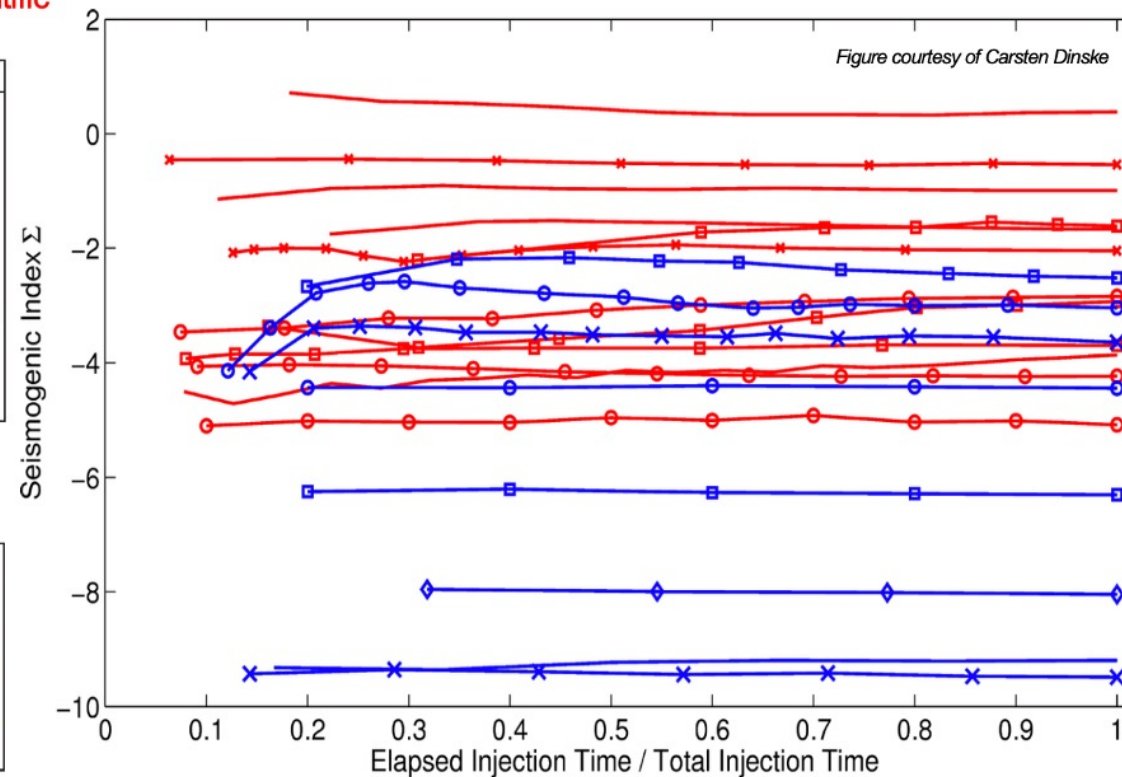
$$\Sigma = \log_{10} \left[ \frac{\zeta}{C_{max} S} \right] + a_p$$

### EGS, disposal and scientific (crystalline basement)

location (type)	$\Sigma$
Basel (2006/EGS)	0.4
Cooper (2003/EGS)	-0.9
Ogachi (1991/EGS)	-2.6
Ogachi (1993/EGS)	-3.2
Soultz (1993/EGS)	-2.0
Soultz (1995/EGS)	-3.8
Soultz (1996/EGS)	-3.1
Soultz (2000/EGS)	-0.5
KTB (1994/SCI)	-1.65
KTB (2004-05/SCI)	-4.2
Paradox (1997-2000/WD)	-2.6
Unterhaching (HYD)	-4.8

### Hydraulic Fracturing (sediments)

Barnett Shale (FRAC)	-9.2
Cotton V. A (FRAC)	-6.2
Cotton V. B (FRAC)	-4.4
Cotton V. C (FRAC)	-9.4
Canada, BC, A (FRAC)	-2.9
Canada, BC, B (FRAC)	-2.3
Canada, BC, C (FRAC)	-3.5
Australia (FRAC)	-8.0



# The Seismogenic Index

*Shapiro, Dinske, Langenbruch and Wenzel, 2010*

Injection  
Volume-based:

## Gutenberg-Richter law for fluid injection-induced seismicity

$$\log_{10} [N_{\geq M}(t)] = \underbrace{\log_{10} [V_I(t)] + \Sigma}_{\text{a-value of the classical GR relation}} - bM$$

$$\Sigma = \log_{10} \left[ \frac{\zeta}{C_{max} S} \right] + a_p$$

... reformulated for **incorporating multi-physics modelling** of pore pressure and stress changes:

$$\log N_{\geq M}(t) = \Sigma + \log_{10} \left[ \int \frac{S M(\Delta CFS(t))}{\sin(\varphi)} dV \right] - bM = \Sigma + F_{CFS}(t) - bM = a_{IS}(t) - bM. \quad (4)$$

*Cacace et al., 2021, Shapiro et al., 2010, Langenbruch et al., 2024*

Here, the fluid volume has been replaced by the integral of the change of the Coulomb Failure Stress ( $\Delta CFS(t)$ ) over the entire reservoir domain (Cacace et al., 2021). In Eq. (4),  $S$  is the uniaxial storage coefficient,  $\varphi$  is the friction angle, and  $M(\Delta CFS(t))$  is the minimum positive monotonic majorant of  $\Delta CFS(t)$ . Eq. 4 can be used in combination of a multi-physics-based model computing the change of  $\Delta CFS$  in space and time.

# Physics-based truncation of the Gutenberg-Richter law and the Seismogenic Index

Seismogenic Index model:

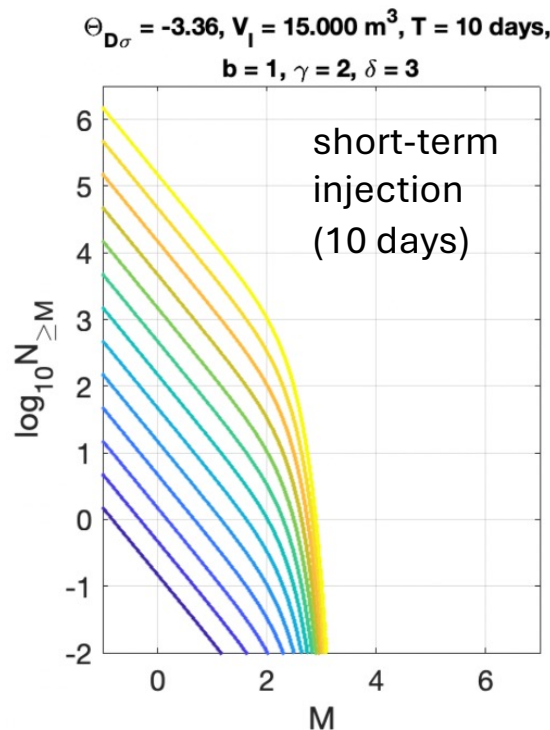
$$\log_{10} [N_{\geq M}(t)] = \log_{10} [V_I(t)] + \Sigma - bM$$

Exponential taper:

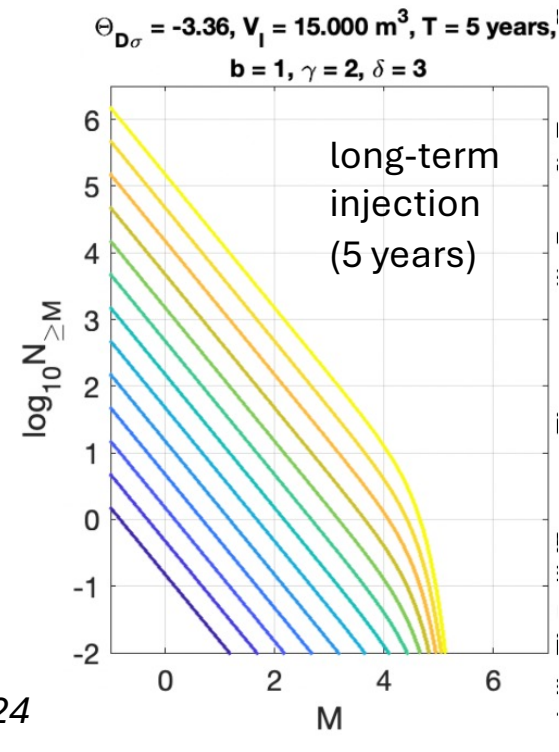
$$P(M) = \beta \left( e^{-\beta(M - M_c)} - \gamma e^{\delta(M - M_m)} \right), M > M_c,$$

$$\beta = \log(10)b,$$

$$M_W^{max} = \log_{10} T + \Theta_{D\sigma}$$



The same fluid volume is injected in both cases!



The maximum magnitude is controlled by the injected volume ( $\Delta CFS$ ), the time and the seismo-tectonic conditions!

We combine the Seismogenic Index model and the Seismic Nucleation Constant.

**Our GR-law includes a volume (CFS) based rate model (a-value) and time-based maximum magnitude ( $M_m$ )!**



# Conclusions

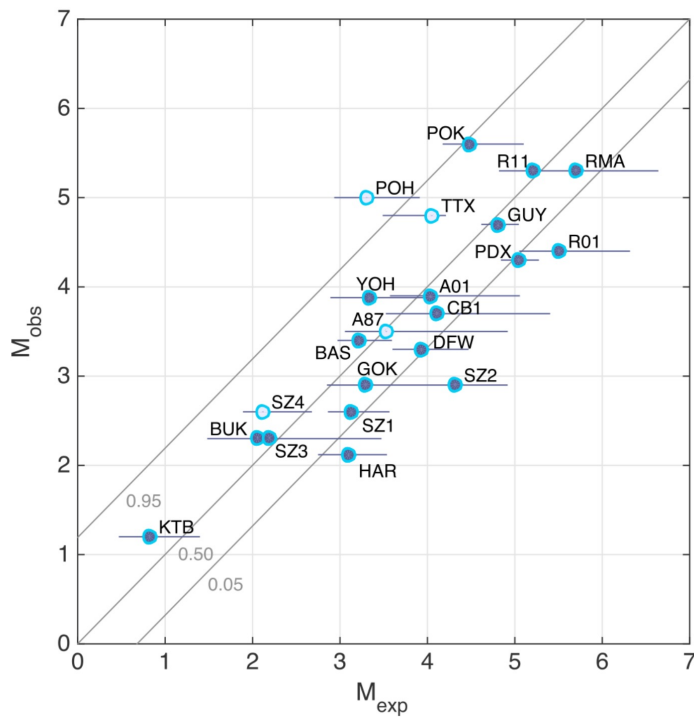
A simple model of pressure diffusion controlled rupture sizes explains the observed increase of maximum magnitudes of induced earthquakes with time.

Our model can be used for truncation of the Gutenberg-Richter law and to understand magnitudes to be included in pre-operational risk and hazard scenario models.

We combined the Seismogenic Index model and the Seismic Nucleation model and reformulated the GR-law. It includes a volume ( $\Delta CFS$ ) controlled earthquake productivity and a time controlled maximum magnitude.

# What controls the maximum magnitude of induced earthquakes?

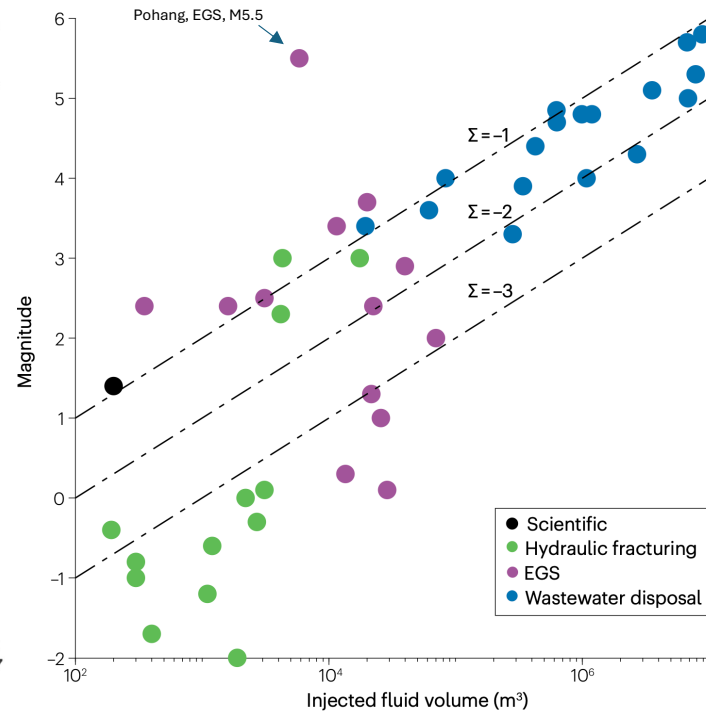
$M_{\max}$  as large as (statistically) expected



van der Elst et al., 2016, JGR

$$\hat{M}_{\max} = M_c + \frac{1}{b} \log_{10} N$$

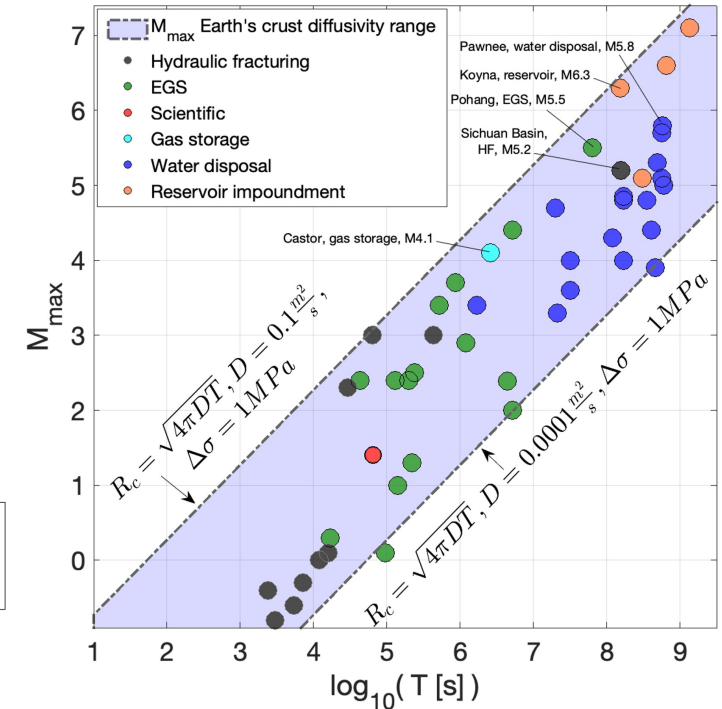
Volume-controlled  $M_{\max}$



Moein, Langenbruch et al., 2023, Nature Reviews Earth & Environment  
Shapiro, Dinske, Langenbruch, Wenzel, 2010, TLE

$$M_{\max}(t) = \frac{1}{b} [\Sigma + \log_{10} V(t)]$$

Time-controlled  $M_{\max}$



Langenbruch et al., 2024, Philosophical Transactions A  
Shapiro et al., 2021, Nature Comm.

$$M_W^{\max} = \log_{10} T + \Theta_{D\sigma}$$

$$\Theta_{D\sigma} = \log_{10} D + \frac{2}{3} \log_{10} \Delta\sigma - 4.7281$$

# Acknowledgements



Thank you!