# Inferring the evolution of the seismicity rate using ETAS model

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## Method: ETAS model (Ogata, 1988)

External force term = background seismicity rate

triggering

$$Earthquake\ rate\ \lambda(t) = \mu_0 + \sum_i K e^{\alpha(M_i - M_c)} \frac{1}{(t + c - t_i)^p}$$

Omori's aftershock law

Estimation

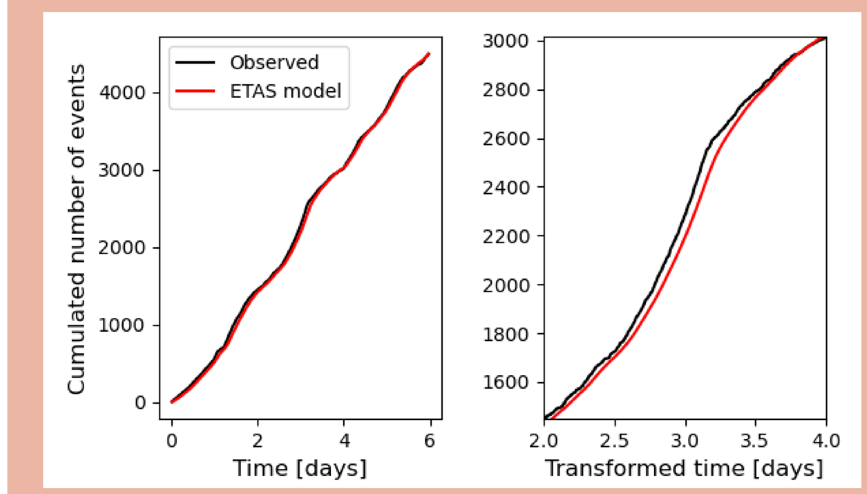
Parameters

$$\theta = (\mu, K, \alpha, c, p)$$

$$logL(\theta) = \sum_{i=1}^{N} \log\left(\lambda(t_i, \theta)\right) - \int_0^T \lambda(t, \theta)dt$$

Maximize logL and use **AIC = 2*nb_param-2*logL** to find best solution

- Use module optimize of Scipy in python
- Use L-BFGS-B algorithm that perform a bound constrained minimization




| Parameter | Value |
|---|---|
| μ | 2.6 |
| K | 0.6 |
| p | 10 |
| a | 8e-6 |
| c | 0.75 |

Fit not optimal, a too low and p too high

Relationship for induced seismicity:

$$Earthquake\ rate\ \lambda(t) = \mu(t) + \sum_i K e^{\alpha(M_i - M_c)} \frac{1}{(t + c - t_i)^p}$$

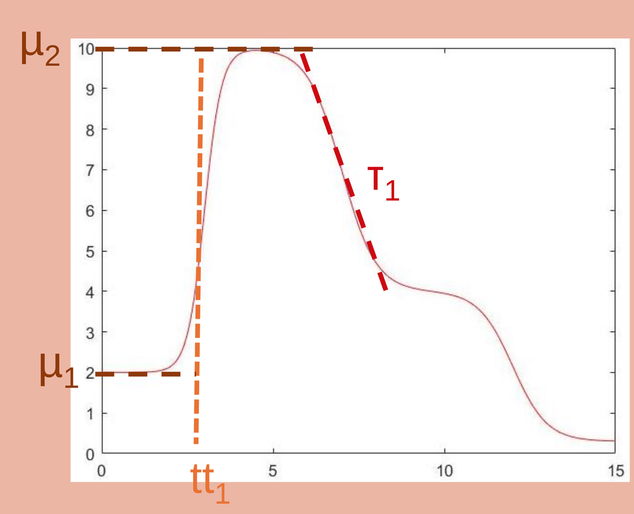
Amplitude of the steps

Time cuts

$$\mu(t) = \frac{\mu_H + \mu_1}{2} + \sum_{i=1}^{H-1} \tanh\left(\frac{t - tt_i}{\tau_i}\right) * \frac{\mu_{i+1} - \mu_i}{2}$$

Smoothness of the variation

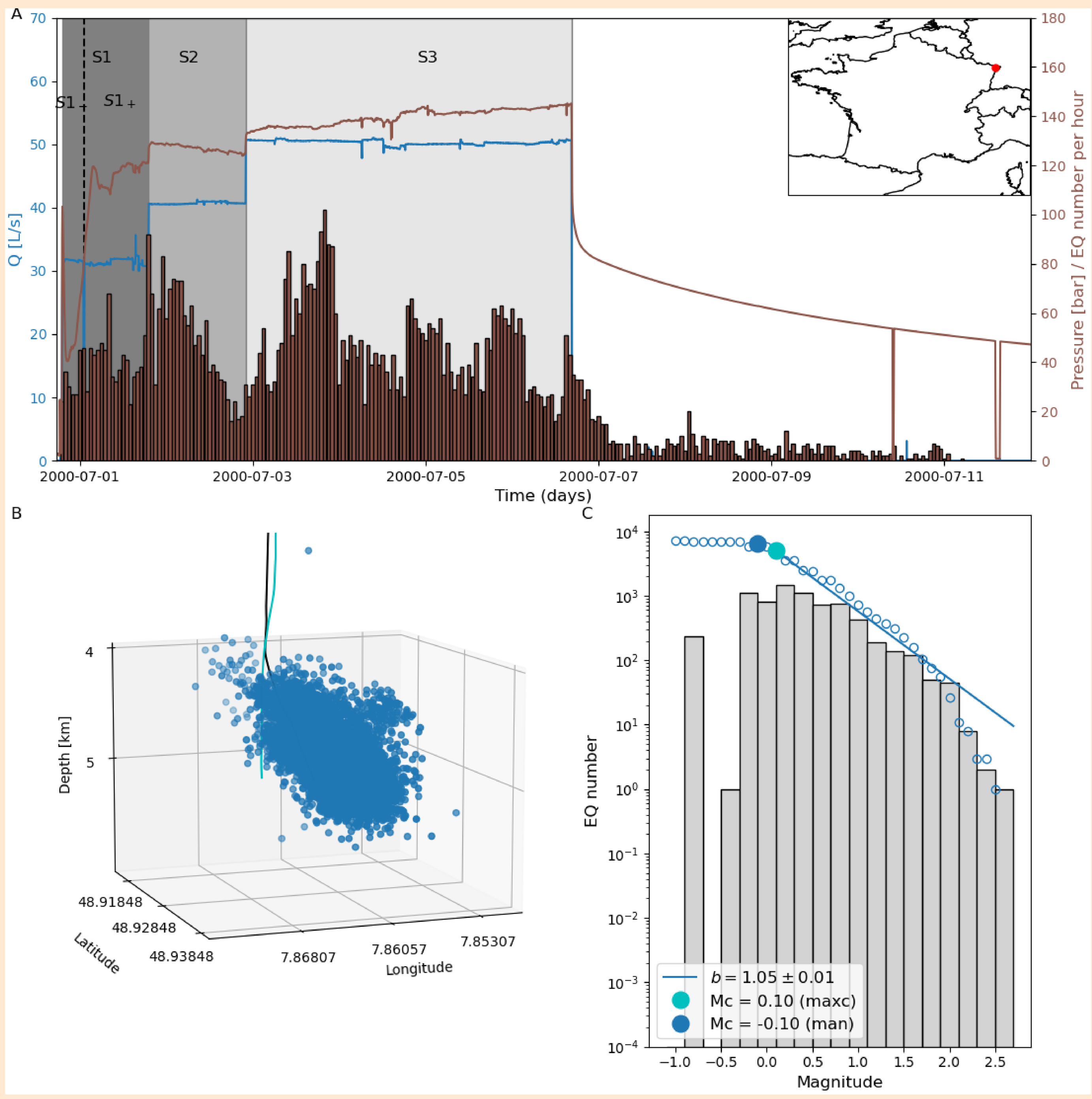
H: number of steps




Assumption 1: triggering term is independent of injection parameters

Assumption 2: μ is related to injection and vary by steps during stimulation

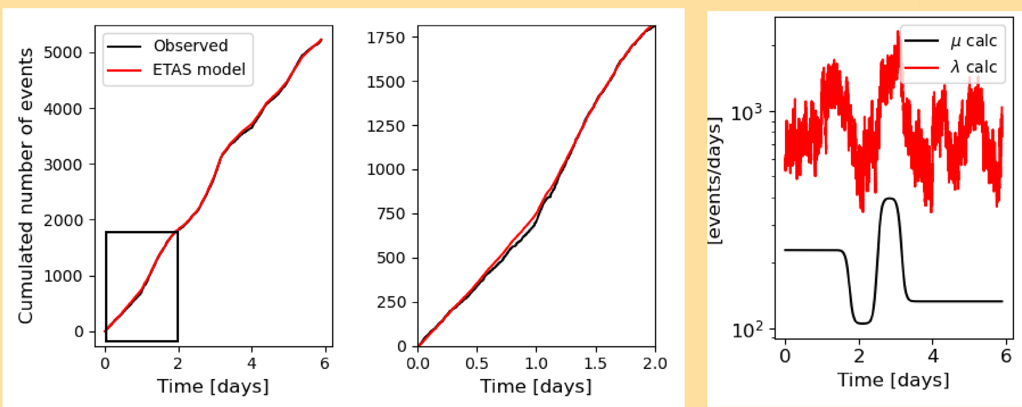
## Data: Soultz-sous-Forêts 2000 stimulation data



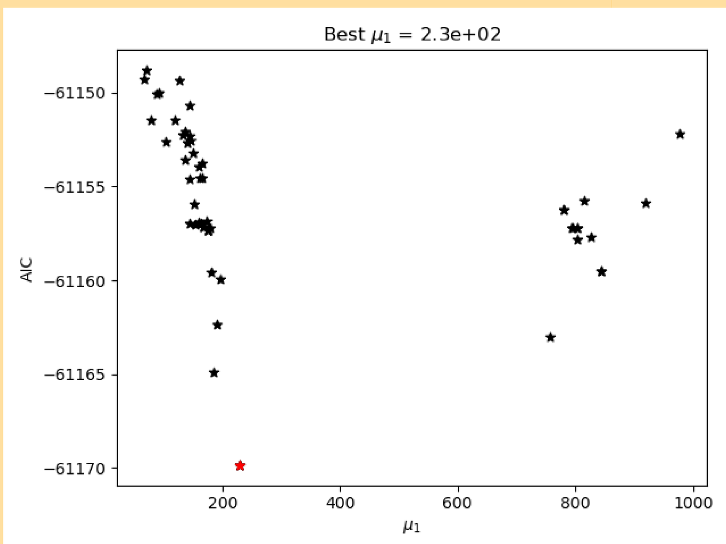

## Results

| Parameter | Value |
|---|---|
| $\mu_i$ | 228/105/397/132 |
| $tt_i$ | 1.73/2.54/3.14 |
| $\tau_i$ (fixed) | 0.1/0.1/0.1 |
| K | 0.02 |
| p | 1.67 |
| a (fixed) | 0.001 |
| c (fixed) | 0.008 |

- Fit is good
- Parameter values are within expected range
- K is low relative to μ: the **background seismicity rate term is dominant**

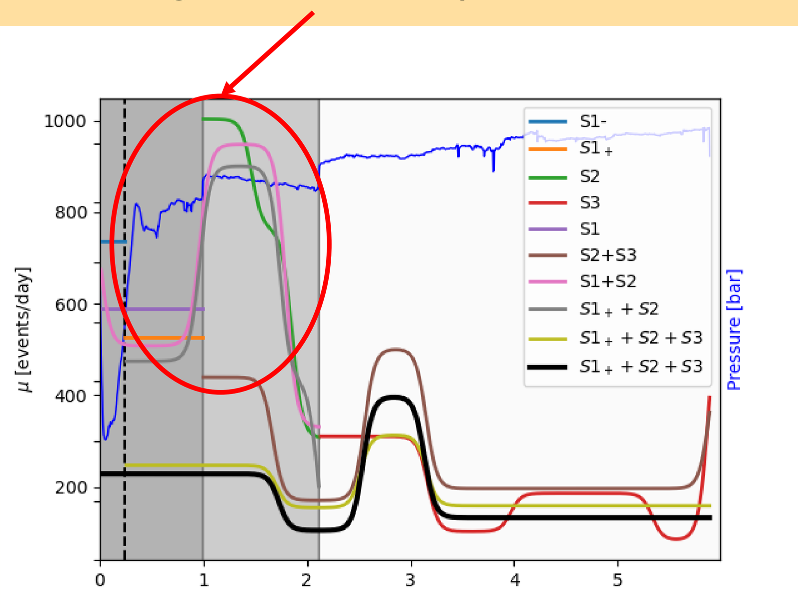



*Fit of ETAS with observations*




*$\mu_1$ in function of AIC for different initial values*

Additional increase in μ + higher values of μ



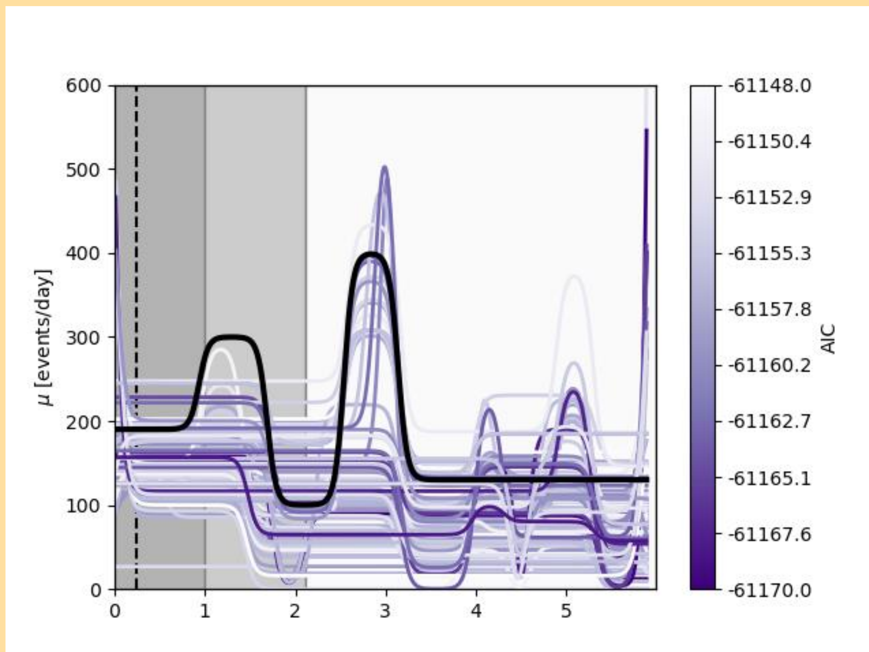

*μ for inversion realised on different time windows*

| Set | K | p |
|---|---|---|
| S1- | 1.0e-6 (bound model) | 0.1 |
| S1+ | 3.2e-5 | 3 (bound model) |
| S2 | 3.4e-5 | 3 (bound model) |
| S3 | 2.1e-2 | 1.68 |
| S1 | 2.1e-5 | 3 (bound model) |
| S2+S3 | 9.9e-3 | 1.9 |
| S1+S2 | 3.6e-5 | 2.9 |
| S1+ +S2 | 3.8e-5 | 3 (bound model) |
| S1+ +S2+S3 | 1.4e-2 | 1.8 |

Two kinds of solutions:

1. μ ~100 evts/day, K~1e-2, p~1.5
2. μ ~700 evts/day, K~1e-5, p~3

2. is less likely: produced with less events and p value is high compared to usual values
2. means no triggering term, all seismicity is explained by background seismicity rate



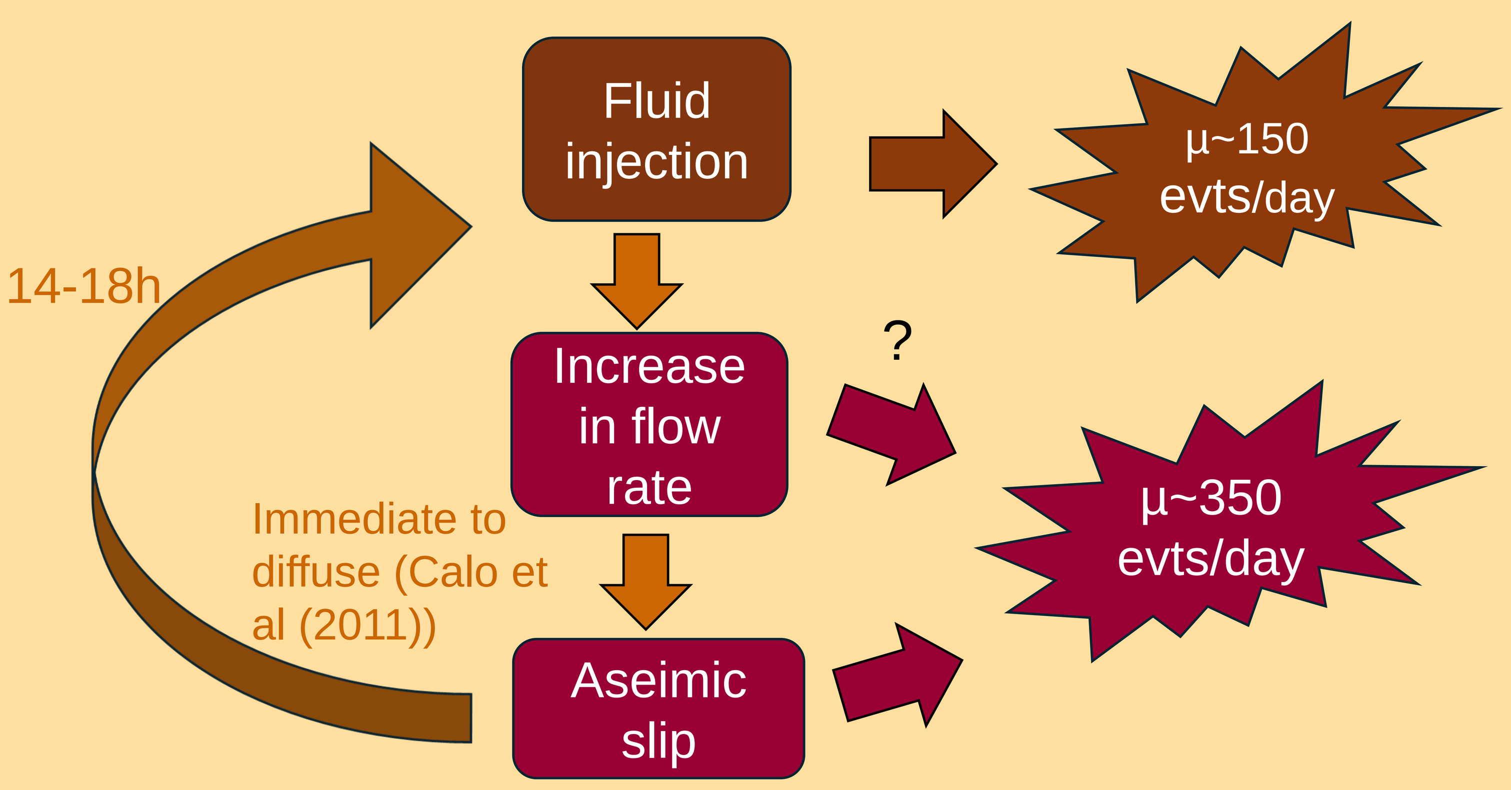

*All solutions for inversion over the whole catalogue, best solution in black*

Redo the inversion using 4 tanh and initial value $tt_1$=0.96

Find lowest AIC of all inversions

Need better scheme to sample model space, use more initial values

## Interpretation




## Conclusion

The ETAS method with a **time dependent μ is promising** for analyzing induced seismicity. Results shows that a bias in μ value is introduced if a subset is used and even if all variations are not reproduced **using the whole catalogue, tendance are found, and results seems more reliable**.
To have a more robust result more initial values must be tested.
Nevertheless, the results allows to interpret the seismicity evolution in regard to the injection evolution. The seismicity behavior may be interpreted as the **seismicity being driven by fluid injection as well as aseismic slip** that occur in response to injection increase (Calo et al, 2011).

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