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Forecasting induced earthquake magnitudes using extreme value theory

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Method

Upper limit and record-breaking event theory (Cooke, 1979):

$$M_{UL} = 2M_n^O - \sum_{i=0}^{n-1}\left[\left(1-\frac{i}{n}\right)^n - \left(1-\frac{i+1}{n}\right)^n\right]M_{n-i}^O$$

- *M* can be anything – in this case earthquake magnitude **or** moment
- $M^o{}_i$ are the observed values, ordered from smallest to largest (so $M_n{}^o$ is the largest observed event magnitude).

Method

Or…

- … estimate the size of the next "jump"

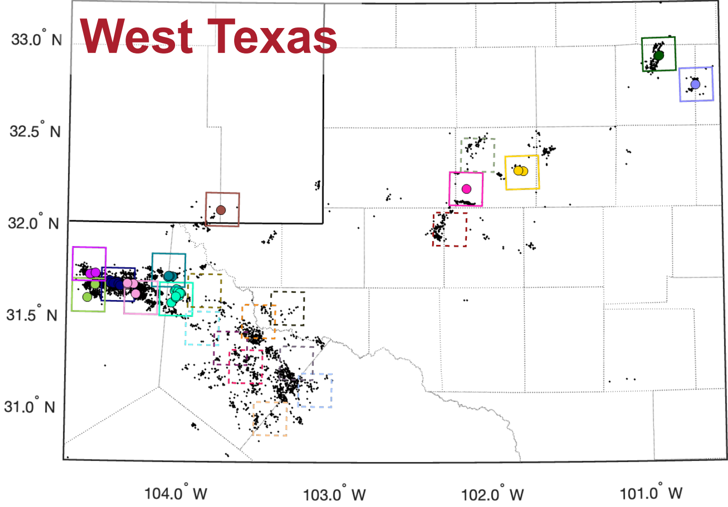
$$M_{JL} = M^{O}_{MAX} + \Delta M_{MAX}$$

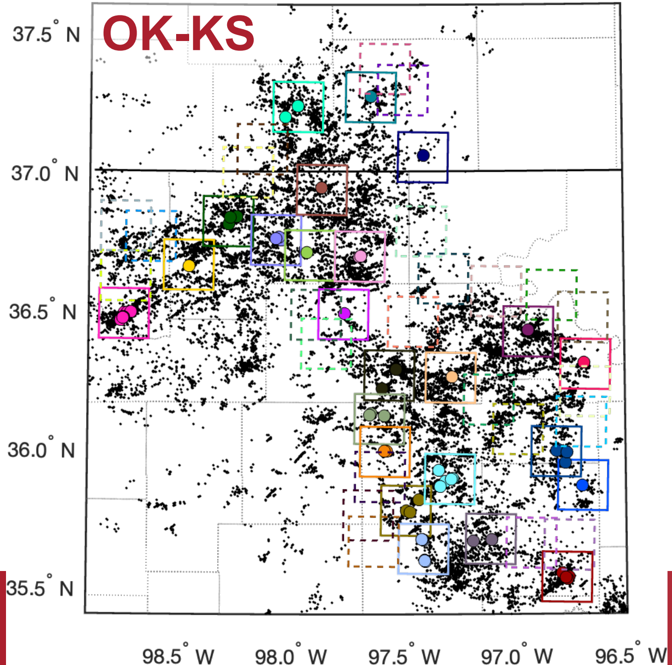
$$\Delta M_{MAX} = 2\Delta M^{O}_{n_j} - \sum_{i=0}^{n_j-1}\left[\left(1 - \frac{i}{n_j}\right)^{n_j} - \left(1 + \frac{i+1}{n_j}\right)^{n_j}\right]\Delta M^{O}_{n_j-i}$$

- ΔM^{o}_{i} are the observed jumps (increase w.r.t any previous event), ordered from smallest to largest (so ΔM^{o}_{n} is the largest observed event magnitude jump).

Datasets

- **86 individual sequences…**
- … containing **331 record breaking** events

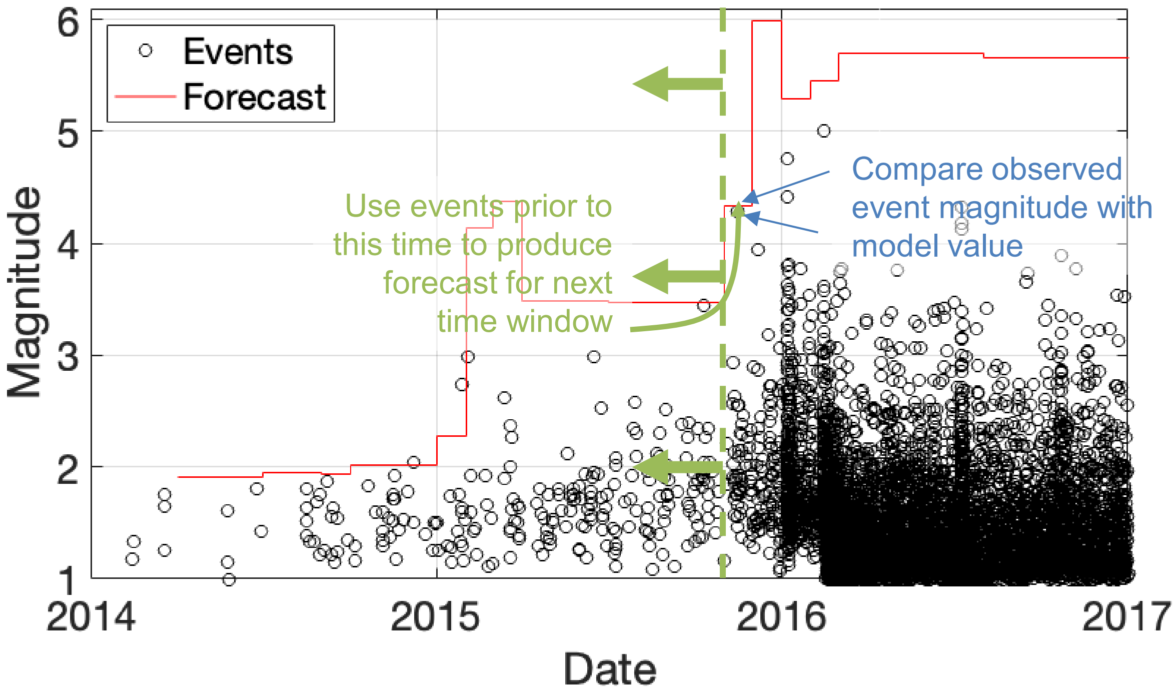




Model Performance

We make forecasts in a pseudo-prospective manner:

- At a given time, all previous events are used to estimate M_{NRB}
- If the next window contains a new "record-breaking" event (M^{O}_{NRB}) then we compare the observed event magnitude with the modelled values

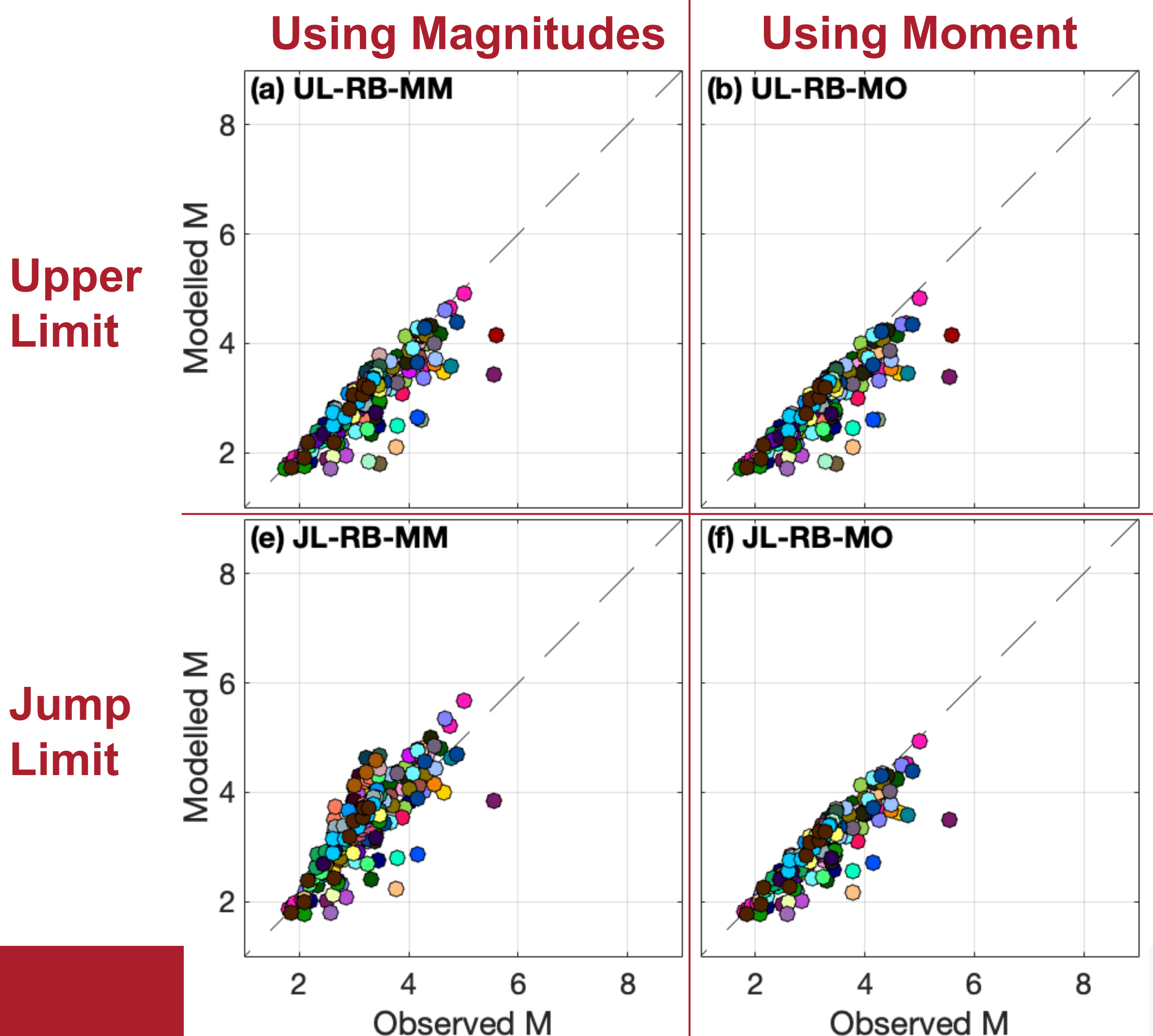


Performance Metrics:

- RMS error, $\boldsymbol{\sigma_{RMS}}$; Pearson correlation coefficient, ***r***; Line of best fit between modelled and observed mags, ***m***; How many significant underpredictions, $\boldsymbol{N_{UP}}$ [$M_{OBS} > M_{MOD} + 0.5$]

Results

- Forecasts generally work well: metrics are better than for S_I or S_{EFF} models (see Verdon et al., PTRSA, 2024)
- Jump-limited using magnitudes is more scattered (lower r value, higher σ_{RMS}.
- Number of underpredictions is unsatisfactory (10 – 25 %)



An Accidental Solution…

- From Mendecki (2016):

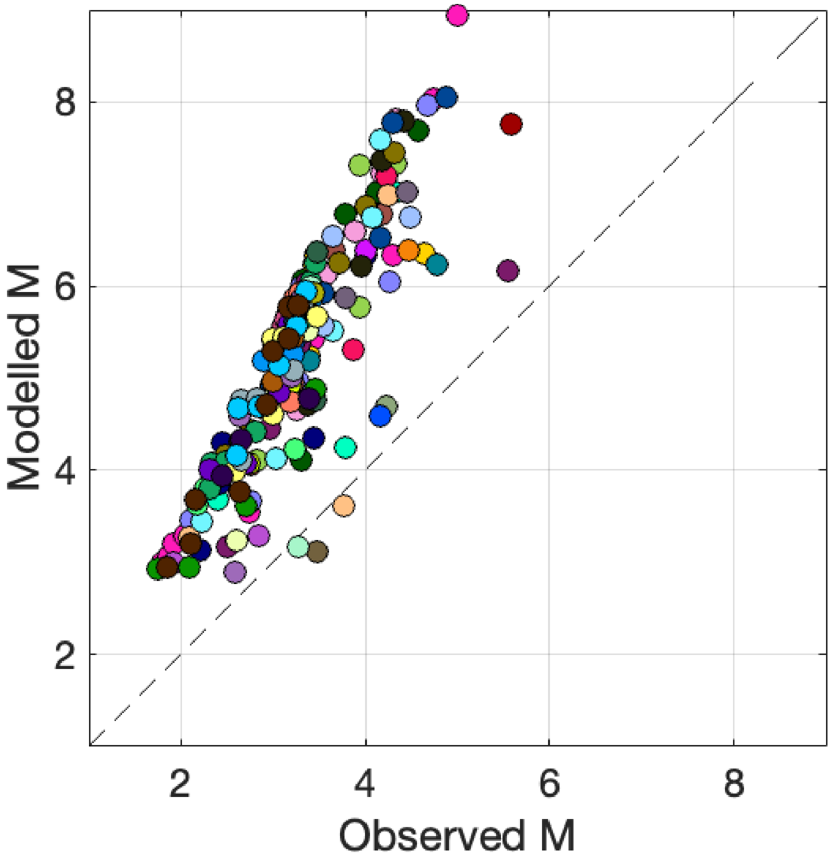
$$P_{max} = 2P_{maxo} - \sum_{\boxed{i=1}}^{n-1} P_{maxo-i}\left[\left(1-\frac{i}{n}\right)^n - \left(1-\frac{i+1}{n}\right)^n\right],$$

typo!

- Missing *i* = *0*-th term from the summation
- Results generally overpredict, though in a consistent way.

But:

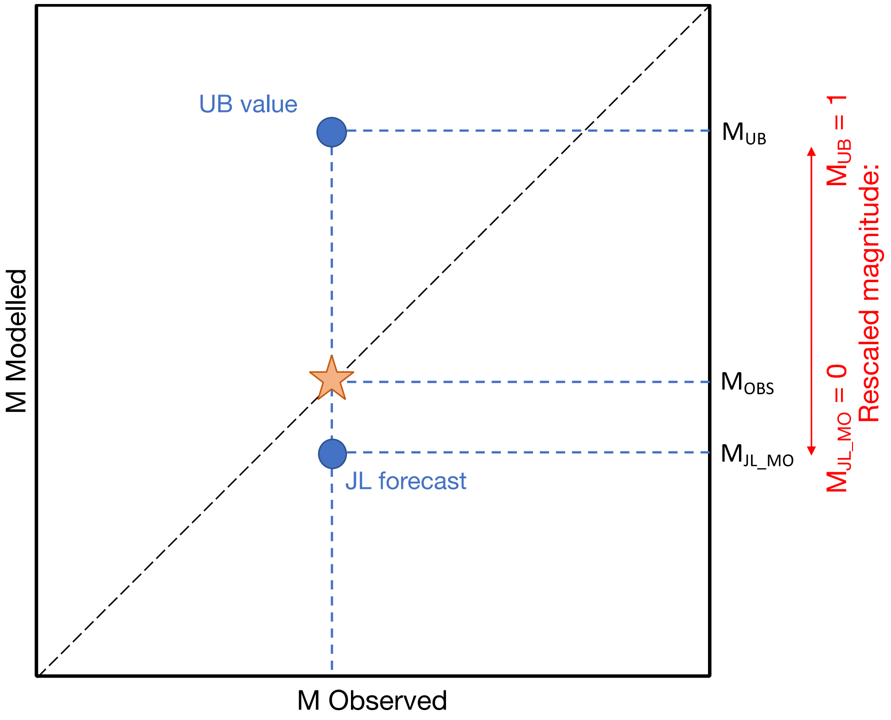
- No overpredictions (in 86 individual sequences, with 330 individual record-breaking events)
- Summation from *i* = 1 defines our upper bound, M_{UB}



Towards an empirically-constrained model

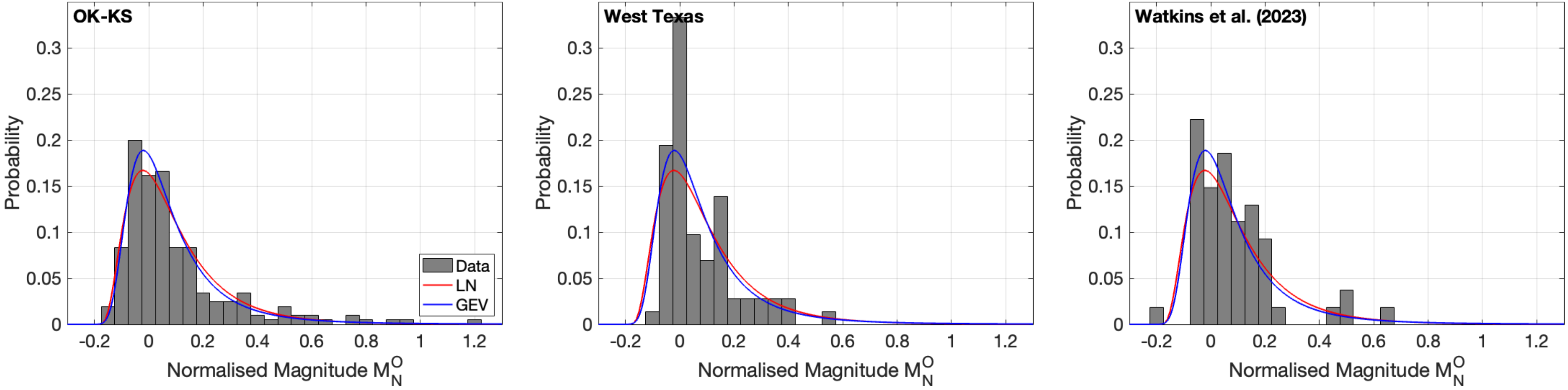
- We normalise each event by the M_{UB} and M_{JL_MO} values at the time the event occurred:

$$M_N^O = \frac{M_{NRB}^O - M_{LB}}{M_{UB} - M_{LB}}$$



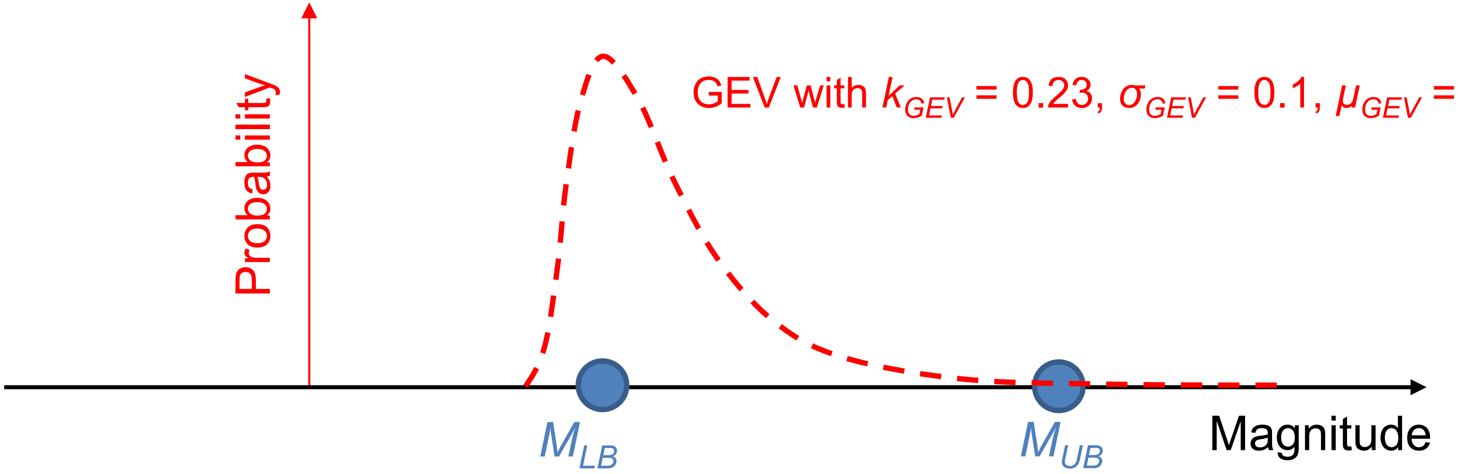
Towards an empirically-constrained model

- Behaviour is consistent – most cases clustered around 0 (i.e., $M^{O}_{NRB} \approx M_{LB}$), but a tail of events reaching towards 1 (i.e., $M^{O}_{NRB} = M_{UB}$).
- Observed distribution is well-fitted by a Generalised Extreme Value (GEV) model (k_{GEV} = 0.23, σ_{GEV} = 0.1, μ_{GEV} = 0.0)



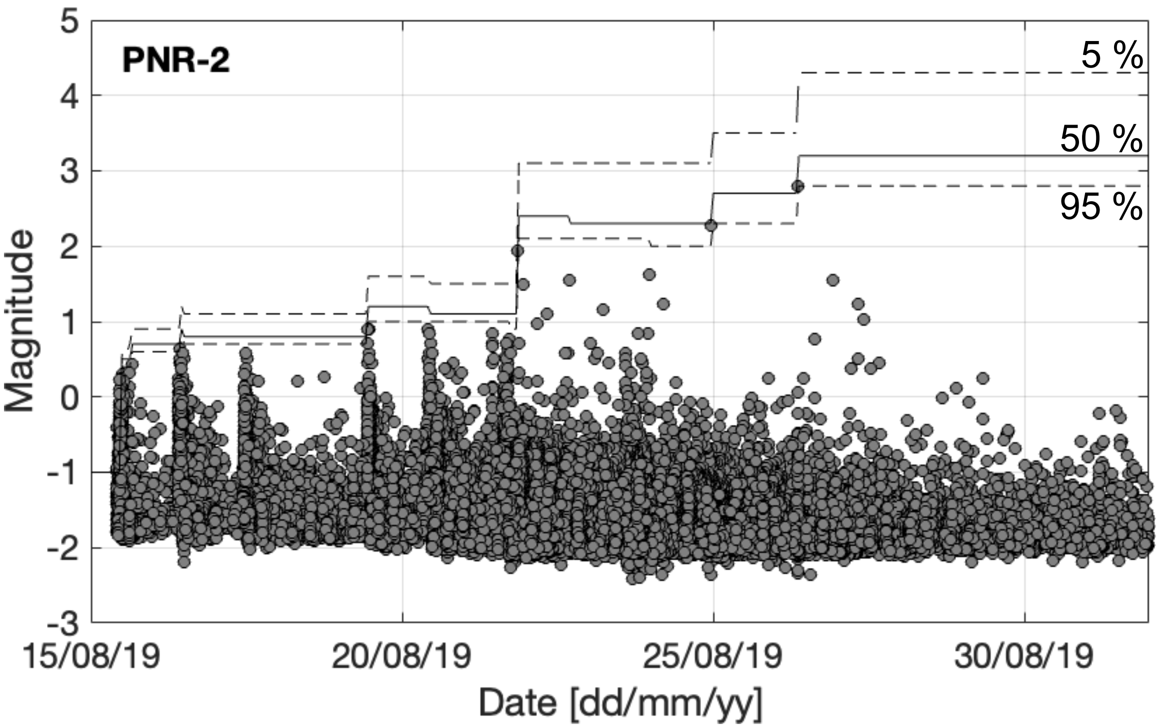
Towards an empirically-constrained model

- We can use this behaviour to produce a probabilistic estimate for the next record-breaking event during a sequence.
- Compute M_{LB} *and* M_{UB}, and assign probability values to magnitudes between these values:



Towards an empirically-constrained model

- **Demo:** Application to PNR-2 – an out of sample case that was not used to define the probability distribution



Conclusions

- M_{MAX} forecasting using extreme value statistics shows significant potential
- We have tested various implementations of this approach across a very large number of case studies. We find good correlation between forecast and observed maximum magnitudes
- We (accidentally) identified a way to deal with outlying underpredictions…
- Empirical calibration with a large number of observed sequences allows us to define a probabilistic estimate for M_{NRB}

Acknowledgements

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