# Groningen source model: refinement, inference and validation

Dirk Kraaijpoel, Sander Osinga, Frans Aben, Bouko Vogelaar, Maarten Pluymaekers

*TNO Geological Survey of the Netherlands* dirk.kraaijpoel@tno.nl

## Background

The seismic hazard and risk analysis of the Groningen gas field as performed by TNO[1] is built on the foundation of the seismic source model developed by Stephen Bourne and Steve Oates (B&O)[2] in the past decade.
The B&O model appears to have struck a sweet spot in terms of the detail of inputs and model complexity, as in the meantime no substantially different model alternatives have been proposed that come close to matching the B&O model in terms of spatial and temporal forecast performance, despite various efforts.

## Refinement

We discuss four refinements to the B&O seismicity rate model

1. Bayesian inference of stress model parameters
2. Replacement of the *rmax* stress model parameter. Fault segments are filtered on their throw-thickness ratio *r* by a low-pass filter with an abrupt cut-off of rmax. It is replaced by a smooth filtering algorithm based on Bernstein polynomials, that allows for n+1 weight groups of fault segments for an expansion of degree n.

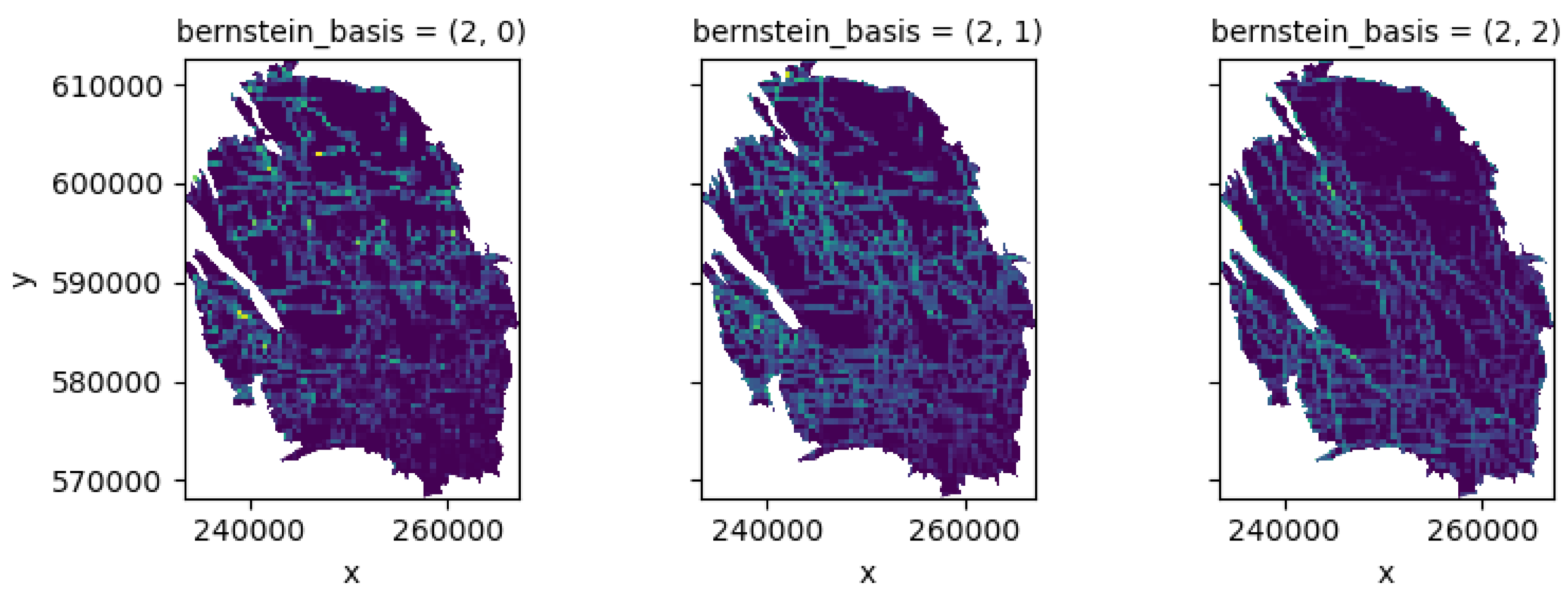



Figure 1: Bernstein expansion of degree 2 (bs2). The colours of the fault traces represent the weights of fault segments in each groups. Smaller ratios have higher weights to the left and lower to the right and vice versa.

3. Introduction of inelastic (RTiCM) stress model: see also the presentation by S. Osinga in the current session
4. Model weighting by means of stacking based on LOO-CV

## Inference

Similar to B&O[2] we perform Bayesian inference on the model parameters and use the posterior predictive for forecasts. However, relative to their work we extend the inference to parameters representing the stress model. In its original form this concerned three parameters that were originally fixed based on optimization: (1) *rmax*, discussed above, (2) *sigma*, representing a smoothing length scale, and (3) *Hs*, representing an elastic modulus.

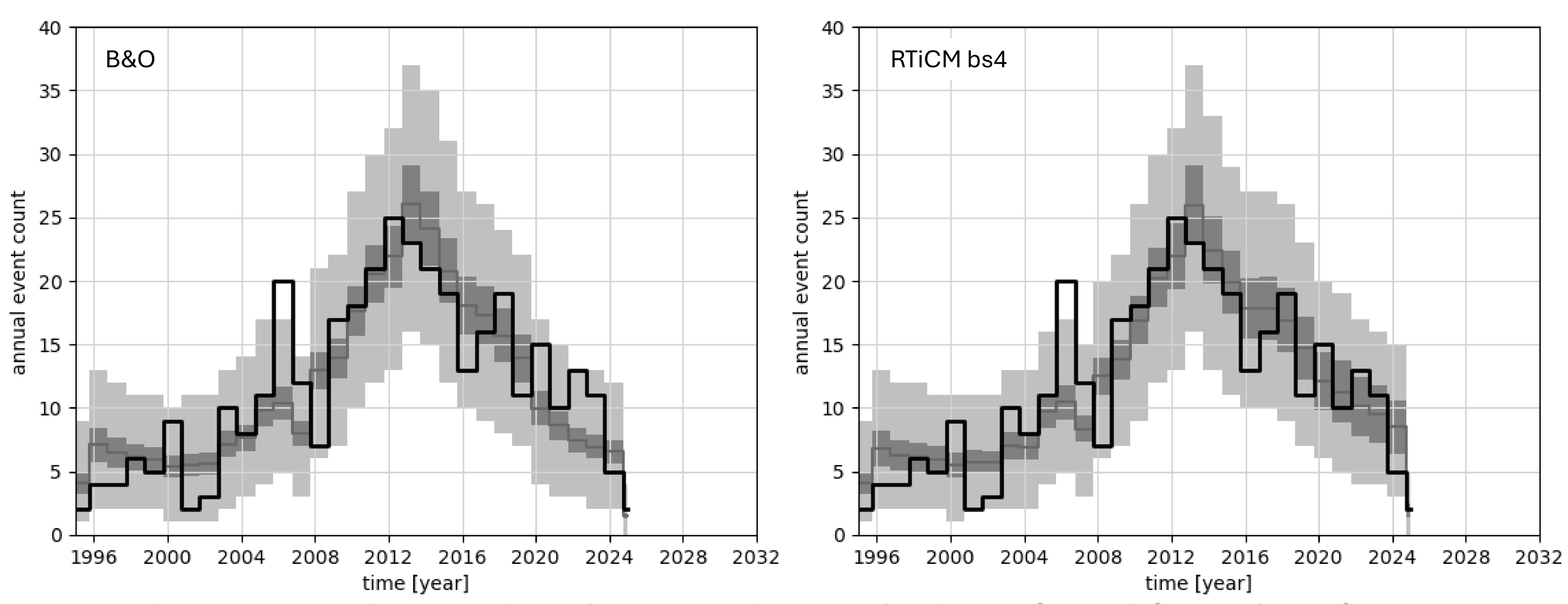



Figure 2: Hindcast using the posterior predictive inferred from data from the period 1995-2021. Left: the original B&O model, right: model with Bernstein expansion of fault weights of degree 4, and RTiCM stress model.

## Validation

For validation we turn to both CSEP-style pseudo-prospective testing (e.g. Rhoades[3]) for significance tests, and Leave-One-Out Cross-Validation (LOO-CV, e.g., Vehtari[4]) for model comparison. The latter also allows for model averaging using stacking (Yao[5]).

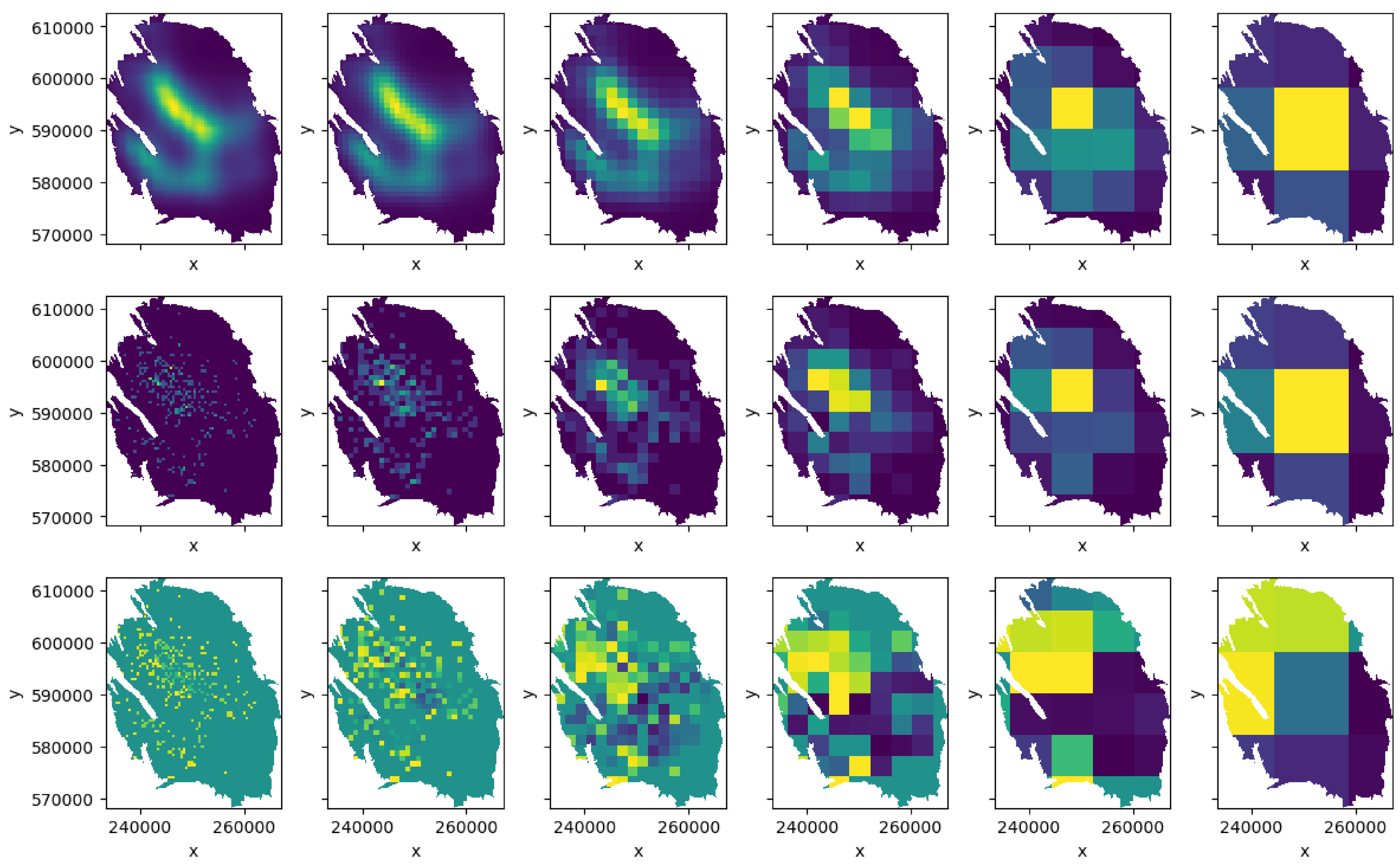


Figure 3: Spatial retrospective tests of the B&O model on different resolutions from left to right. Top row: modelled rates, middle row: observations. Bottom row: Poisson percentiles [yellow=high=more observed then expected, blue=low, less observed than expected]. Clustering of similar shades of yellow/blue in bottom row indicates spatial shortcomings in the model.

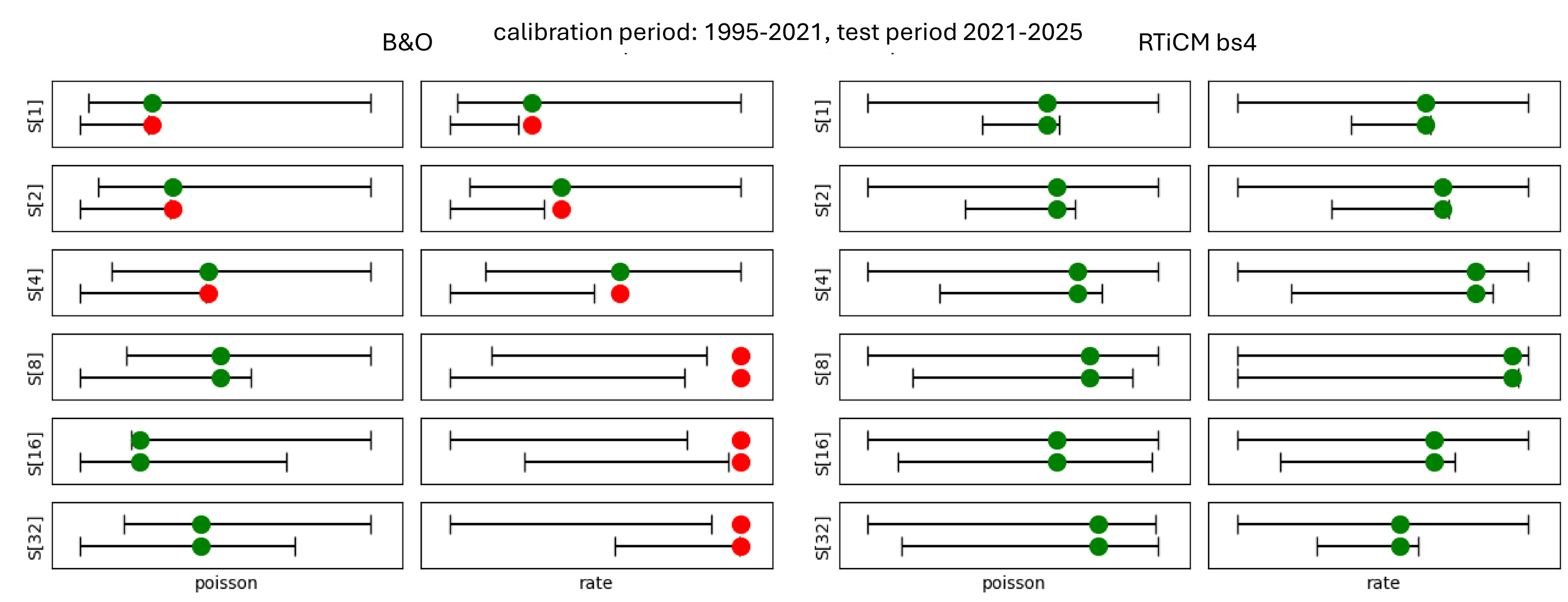
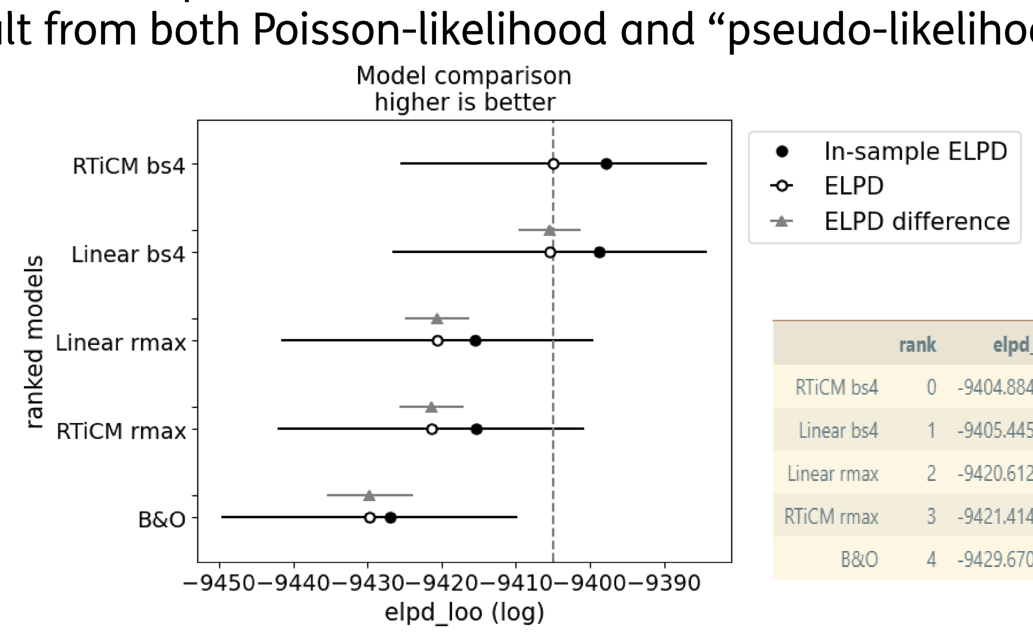



Figure 4: Spatial pseudo-prospective test for B&O (left) en RTiCM bs4 models. Top to bottom: spatial scales. Two bars indicate true-rate and normalized-rate analyses. Dots indicate position of observations relative to 95% modelled likelihood ranges. Result from both Poisson-likelihood and "pseudo-likelihood" (rates) are shown.




| | rank | elpd_loo | p_loo | elpd_diff | weight |
|---|---|---|---|---|---|
| RTiCM bs4 | 0 | -9404.884854 | 7.130403 | 0.000000 | 0.516484 |
| Linear bs4 | 1 | -9405.445913 | 6.642166 | 0.561059 | 0.483516 |
| Linear rmax | 2 | -9420.612416 | 5.152870 | 15.727562 | 0.000000 |
| RTiCM rmax | 3 | -9421.414376 | 6.085924 | 16.529521 | 0.000000 |
| B&O | 4 | -9429.670412 | 2.727406 | 24.785558 | 0.000000 |

Figure 5: Comparison of a few model variations using LOO-CV (PyMC, Arviz). ELPD stands for "expected log pointwise predictive density" (Vehtari[4]). Stacking weights based on 1995-2025 data lead to roughly 52%-48% weighting of RTiCM and linear stress models, with 4th degree Bernstein expansion of fault database. Models without Bernstein weighting are rejected.

## Conclusion

We have improved the predictive performance of the B&O model using a number of refinements, most notably the Bayesian inference of stress parameters and an improved weighting of the contribution of faults segments.

## References

1. TNO 2023 R10682 - Public seismic hazard and risk analysis 2023
2. Bourne and Oates (2017), Extreme threshold failures [...], JGR 122
3. Rhoades et al. (2011), Efficient testing [...], Acta Geophysica 59
4. Vehtari et al. (2017), Practical Bayesian model evaluation [...], Stat Comput 27
5. Yao et al. (2018), Using stacking to average [...], Bayesian Analysis