Concept of fracture displacement basis functions (FDBF) for fast geomechanical simulations of fractured rock

Giulia Conti[1], Stephan Matthai[2], Patrick Jenny[1]
[1]Institute of Fluid Dynamics, ETH Zurich; [2]Department of Infrastructure Engineering, University of Melbourne

1 Motivation

Geomechanical simulations of fractured rock typically are computationally expensive, but essential for studying aperture changes and seismic activity. Our method presents an efficient approach for simulating the geomechanics of fractures subjected to shearing and tensile opening, assuming linear elasticity for the rock matrix. The method leverages single-fracture shear slip and tensile opening solutions and exploits the linear relationship between maximum slip and the induced stress field. By assuming that the slip profiles remain close to imposed elliptical shapes, the method enables superposition of single-fracture basis functions. [1]

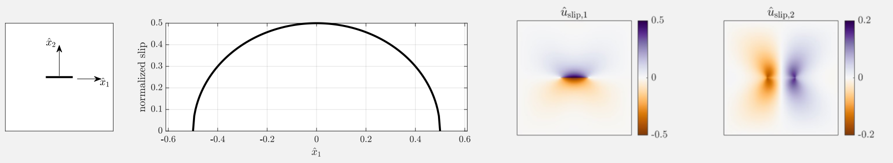
2 Method Overview

Displacement field $\boldsymbol{u}(\boldsymbol{x})$ and stress field $\boldsymbol{\sigma}(\boldsymbol{x})$ are described as a superposition of N weighted fracture specific basis functions (BF) :

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{f=1}^{N} s_t^f \,\hat{\boldsymbol{u}}^f(\boldsymbol{x}) + \boldsymbol{u}^{\infty}(x) \qquad \boldsymbol{\sigma}(\boldsymbol{x}) = \frac{p^*}{L^*}\sum_{f=1}^{N} s_t^f \,\hat{\boldsymbol{\sigma}}^f(x) + \boldsymbol{\sigma}^{\infty}(x)$$

far field influence — far field influence
shear slip — fracture specific BF — ref. pressure / ref. length — fracture specific BF

1. Find normalized BF $\hat{\boldsymbol{u}}_{\mathbf{slip}}(\boldsymbol{x})$ and $\hat{\boldsymbol{\sigma}}_{\mathbf{slip}}(\boldsymbol{x})$ for rock parameters λ and G assuming an elliptical slip profile. Solve for $\boldsymbol{u}$ from $\nabla \cdot \boldsymbol{\sigma} = 0$ and linear elasticity $\boldsymbol{\sigma} = \lambda(\nabla \cdot \boldsymbol{u})\boldsymbol{I} + G(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$. The normalized BF must be computed only once for a specific set of rock parameters.



2. Calculate fracture specific BF as simple transformations of the normalized BF:

$$\hat{\boldsymbol{u}}^f(\boldsymbol{x}) = \boldsymbol{R}\,\hat{\boldsymbol{u}}_{\mathbf{slip}}\left(\frac{\boldsymbol{R}^T(\boldsymbol{x}-\boldsymbol{x}^f)}{L^f}\right) \qquad \hat{\boldsymbol{\sigma}}^f(\boldsymbol{x}) = \boldsymbol{R}\,\hat{\boldsymbol{\sigma}}_{\mathbf{slip}}\left(\frac{\boldsymbol{R}^T(\boldsymbol{x}-\boldsymbol{x}^f)}{L^f}\right)\boldsymbol{R}^T$$

3. Define interaction coefficients from traction and normal influence of stress field $\hat{\boldsymbol{\sigma}}^f$ induced by fracture f on fracture g :

$$\sigma_t^{f\to g} = \frac{p^*}{L^*}\int_{\Omega g} \hat{\sigma}_t^f \, dl \qquad \sigma_n^{f\to g} = \frac{p^*}{L^*}\int_{\Omega g} \hat{\sigma}_n^f \, dl$$

4. Check shear displacement criteria. If $\sigma_t > \mu(\sigma_n - p\;)$, then seek

$$\int_{\Omega g} \sigma_t^g \, dl = \int_{\Omega g} \mu(\sigma_n^g - p^g)\,dl \quad \text{with} \quad \int_{\Omega g} \sigma_{t,n}^g \, dl = \sum_{f=1}^{N} (\, s_t^f \sigma_{t,n}^{f\to g}\,) + \sigma_{t,n}^{\infty\to g}$$

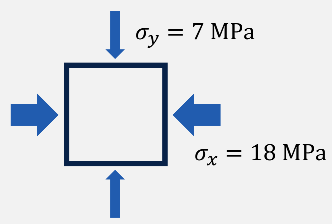
Solve $N \times N$ matrix equation, where N is the number of fractures:

$$\begin{pmatrix} (\sigma_t^{1\to1} - \mu\sigma_{\text{slip},n}^{1\to1}) & \cdots & (\sigma_t^{N\to1} - \mu\sigma_n^{N\to1}) \\ \vdots & \ddots & \vdots \\ (\sigma_t^{1\to N} - \mu\sigma_{\text{slip},n}^{1\to N}) & \cdots & (\sigma_t^{N\to N} - \mu\sigma_n^{N\to N}) \end{pmatrix} \begin{pmatrix} s_t^1 \\ \vdots \\ s_t^N \end{pmatrix} = \begin{pmatrix} f(\sigma_{t,n}^{\infty\to1}, p) \\ \vdots \\ f(\sigma_{t,n}^{\infty\to N}, p) \end{pmatrix}$$

Same procedure for tensile opening, but with criteria $\sigma_t = 0$ and $\sigma_n = p$, which leads to a $2N\times2N$ matrix equation.

3 Simulation Setup

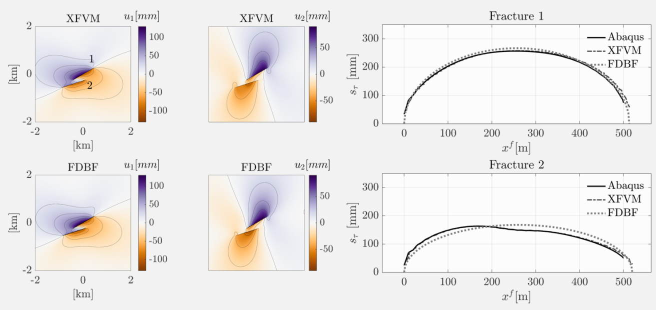
The results are compared to XFVM [2,3] and Abaqus[4] simulations. All simulations are conducted in 2D and assuming far-field compressive stresses $\sigma_x = 18\,\text{MPa}$, $\sigma_y = 7\,\text{MPa}$. The fluid pressure in all fractures is set to $p = 7\,\text{MPa}$.



4 Results

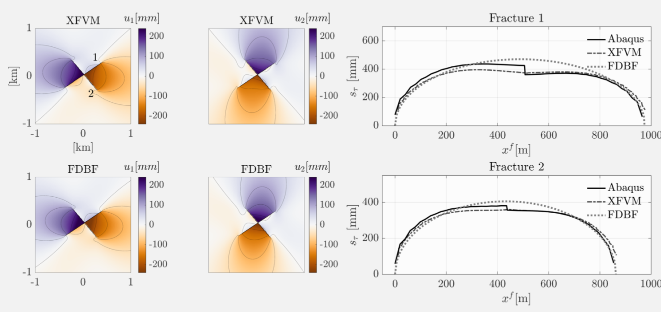
Two non-intersecting fractures

Fracture 2 shows a non-elliptical shear slip profile with XFVM and Abaqus, while FDBF maintains strictly elliptical profiles for all fractures leading to small differences (<5%).



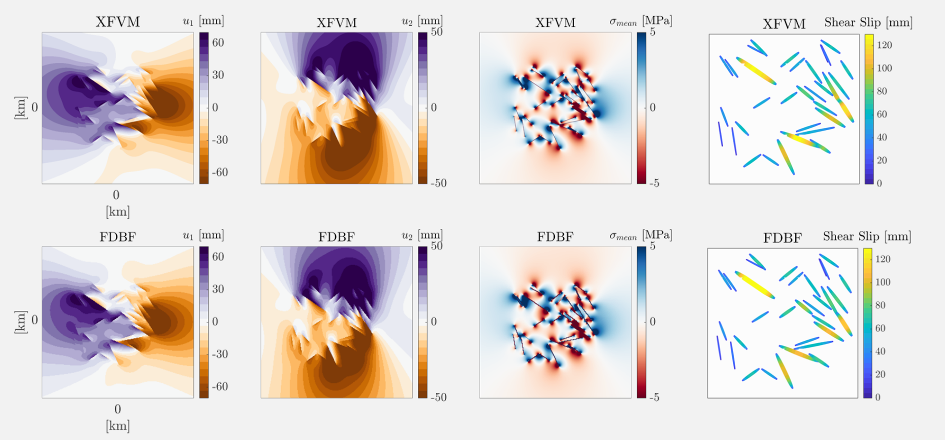
Two intersecting fractures

Intersecting fractures present the most critical challenge for FDBF due to their non-elliptical profiles. Nevertheless, the displacement fields remain similar, demonstrating that FDBF provides a reliable approximation.



33-fracture pattern

The FDBF simulation has just 66 degrees of freedom, whereas XFVM has 8'006'358. It efficiently approximates shear slip, displacement, and stress fields. Once the basis functions are computed, FDBF can evaluate different stress states in seconds.



5 Conclusion

FDBF is an efficient method for calculating shear displacement and tensile opening of interacting fractures.

- Reduces problem to a $2N \times 2N$ matrix equation (N = number of fractures). Promising for large-scale rock displacement simulations.
- Good agreement with Abaqus and XFVM.

Current and future work:

- Accuracy improvement with additional basis functions and degrees of freedom for non-elliptical profiles, e.g., for intersecting fractures.
- Retains efficiency in 3D: 2 degrees of freedom per fracture for shear, 1 for tensile opening.
- Coupling with flow and transport.

References

1. Conti, G., Matthai, S, Jenny, P., 2025, Fracture DisplacementBasis Function (FDBF) Method for Efficient Geomechanical Calculations of Fractured Rock, submitted, SSRN preprint
2. Deb, R., Jenny, P., 2017, Finite volume–based modeling of flow-induced shear failure along fracture manifolds, Computational Geoscience
3. Conti, G., Deb, R., Matthai, S., Jenny, P., 2023, Consistent treatment of shear failure in embedded discrete fracture models using xfvm, Computational Geoscience
4. M. Smith, 2009, ABAQUS/Standard User's Manual, Version 6.9, Dassault Systèmes Simulia Corp