3rd Schatzalp Workshop on Induced Seismicity
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Insights from a closed-form solution for injection- and production-induced stresses in vertical displaced faults

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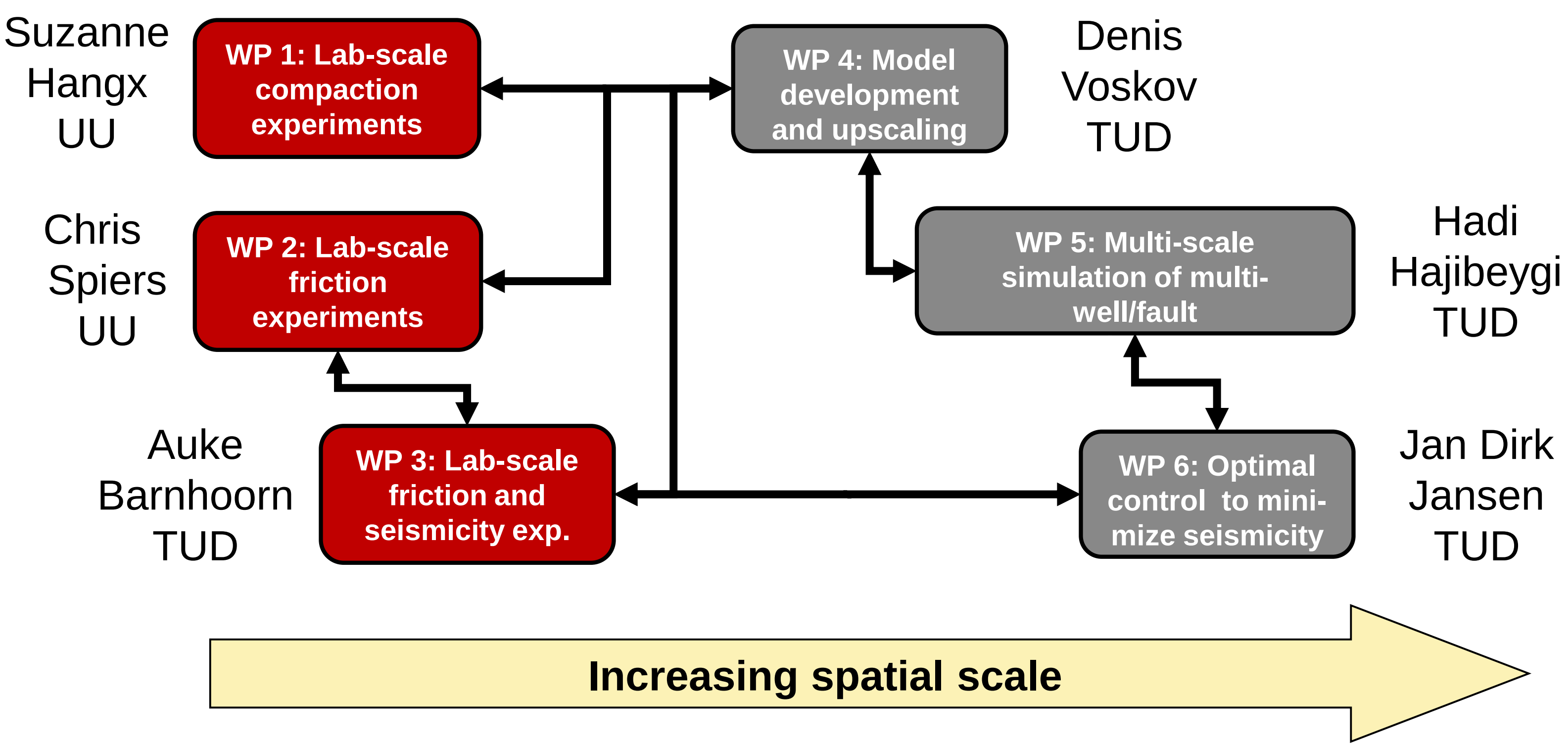
Science4Steer – towards operational control

- Aim: A scientific basis for developing production and reinjection strategies to minimize induced seismicity
- 5-year program at TU Delft and Utrecht University - part of DeepNL
- Combined experimental and numerical approach



Science4Steer – WP 3; Lab Experiments

- Investigator: Auke Barnhoorn @ TU Delft
- Triaxial cell with 30 x 30 x 30 cm blocks
- Induced seismicity resulting from differential pressure and/or pressure decline in displaced faults

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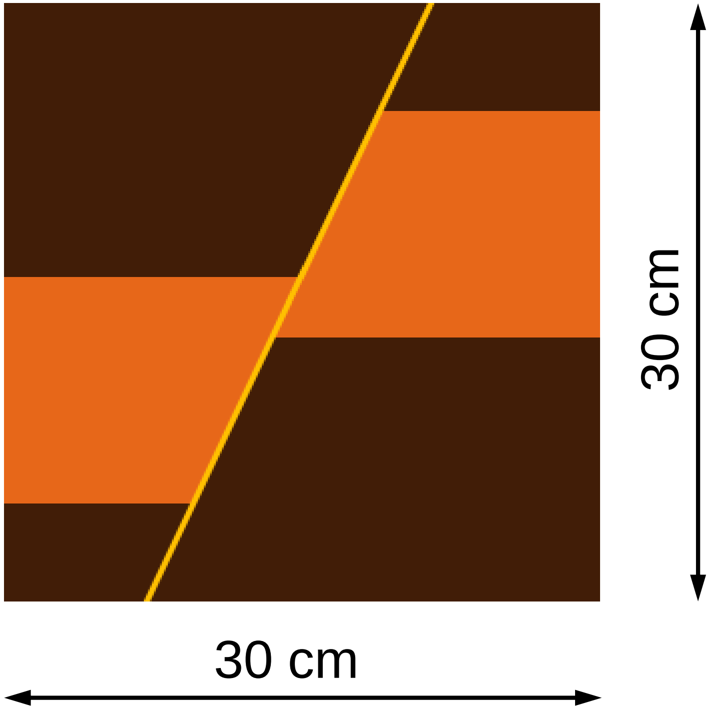

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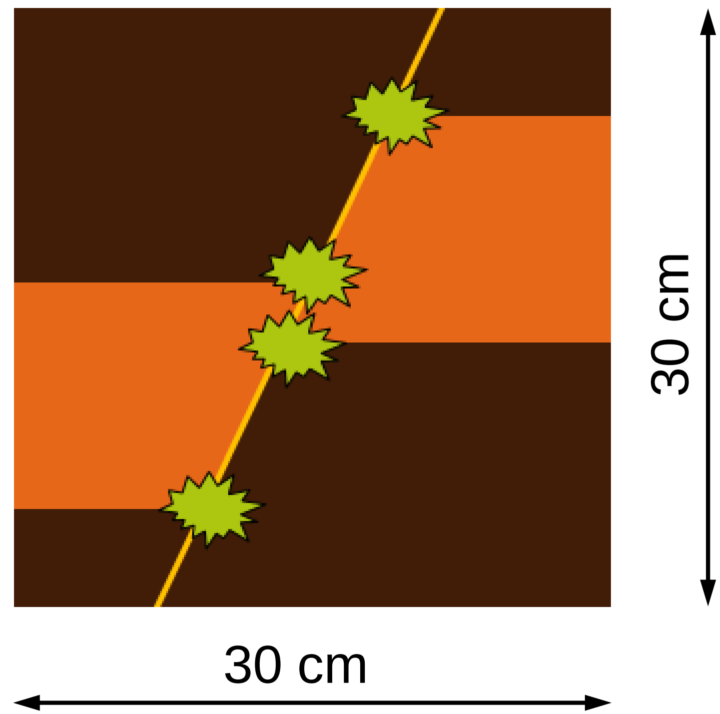
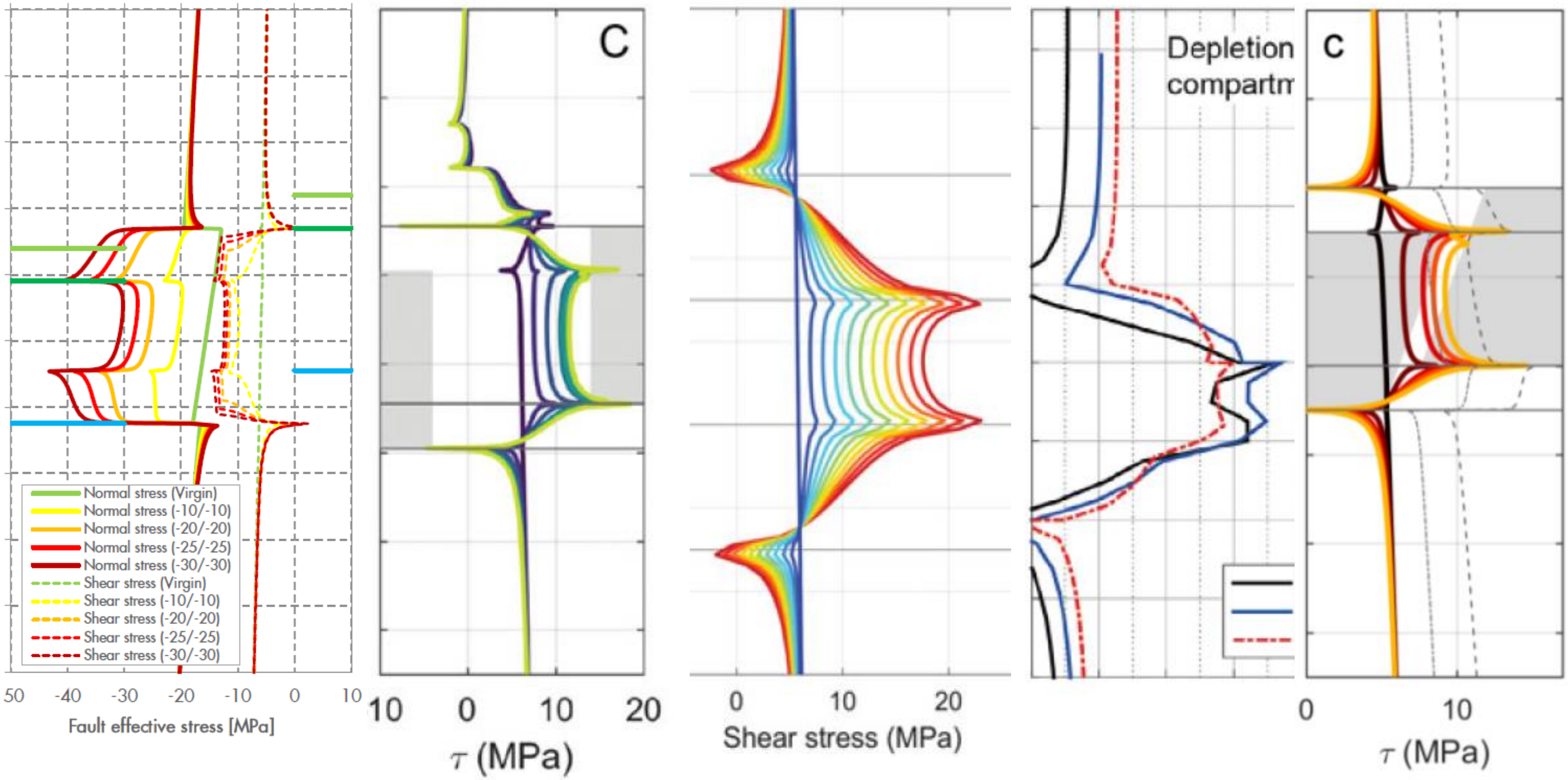
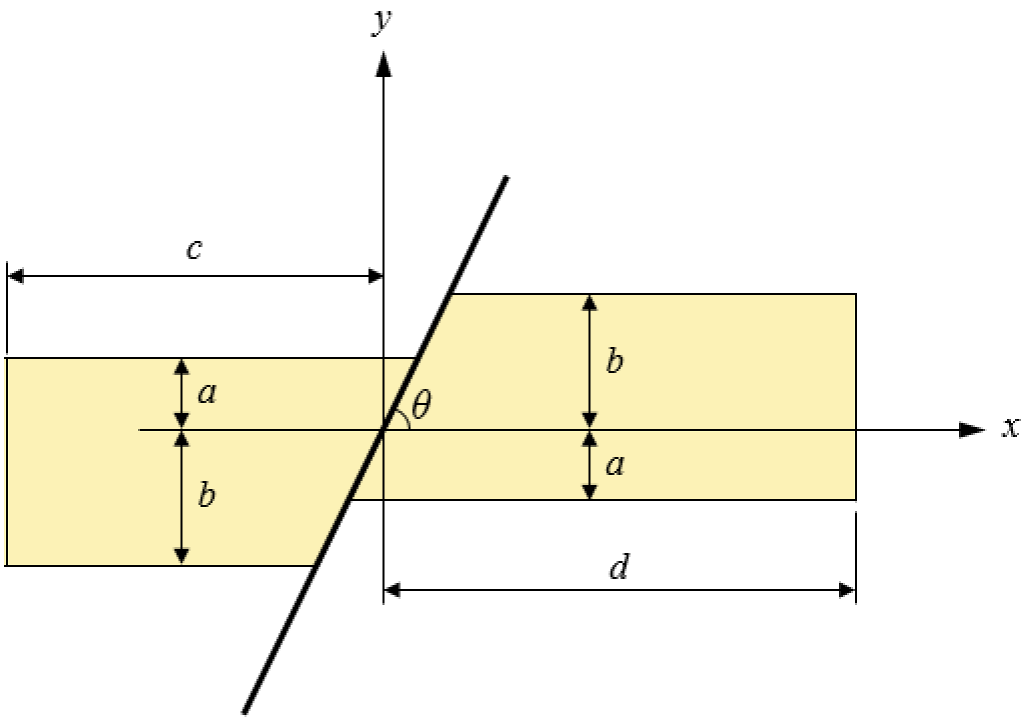


Earlier work – numerical

Roest & Kuilman 1994; Nagelhout & Roest, 1997; Mulders, 2013; Orlic & Wassing, 2013; Orlic et al. 2013; Van den Bogert, 2015; Lele et al. 2016; Zbinden et al. 2017; Bourne & Oates, 2017; Buijze et al., 2017; Van Wees et al., 2017, Haug et al, 2018; Buijze et al., 2019

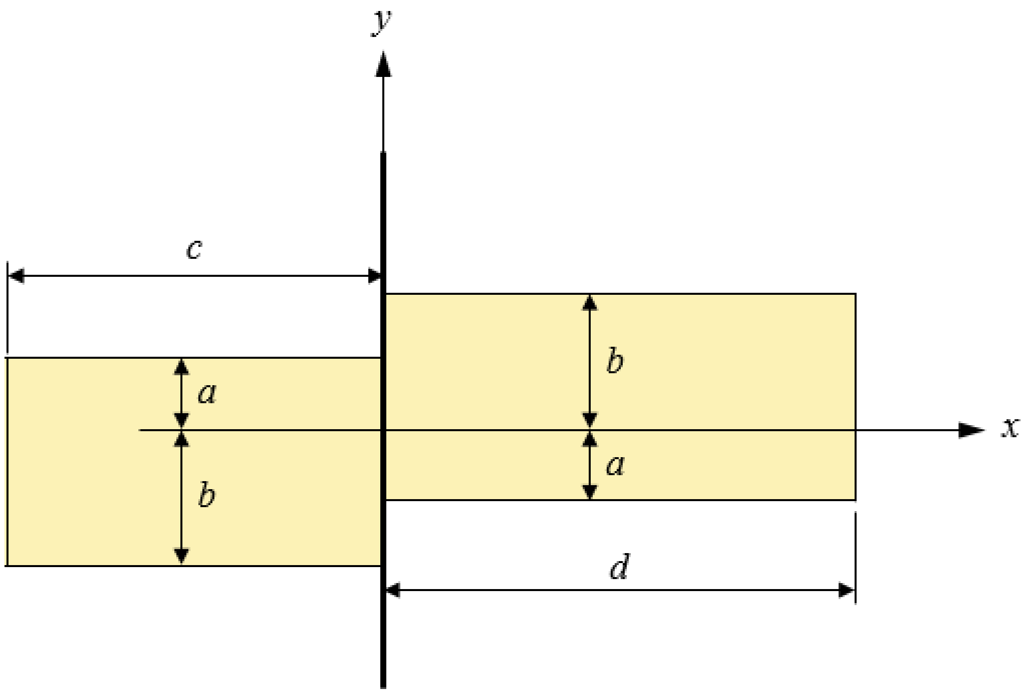


Displaced normal fault model



Fault throw: $t = b - a$
Reservoir height: $h = a + b$
Reservoir width: $w = c + d$
Always overlap: $t < h$

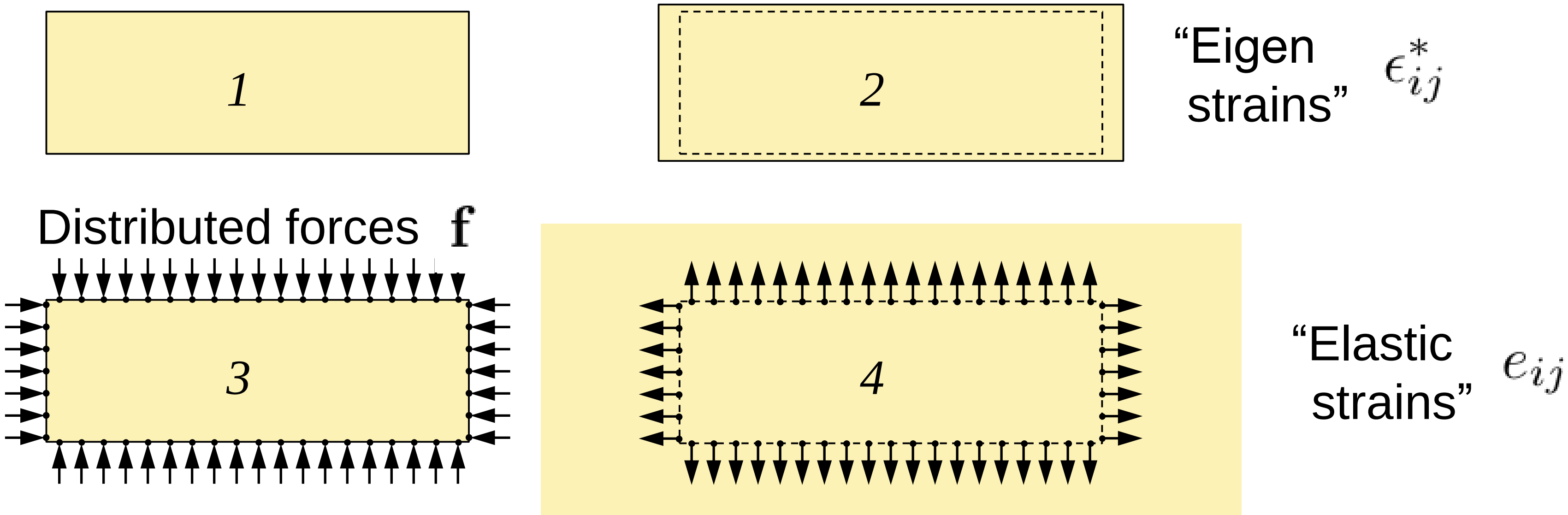
Displaced (normal) fault model – simpler



Infinite, 2D domain
Uniform elastic properties
No transients
Single-phase flow
Straight, zero-width, non-sealing fault

Inclusion theory (1)

Elsheby,1957: cut and weld operation to simulate response to inelastic deformation (e.g. thermal strain, dislocations, pore pressure)



Total strains = elastic strains + eigen strains: $\epsilon_{ij} = e_{ij} + \epsilon^*_{ij}$

For porous media: $\epsilon^*_{ij}\delta_{ij} = \dfrac{\epsilon^*}{3} = \dfrac{\alpha p}{3K}$

Inclusion theory (2)

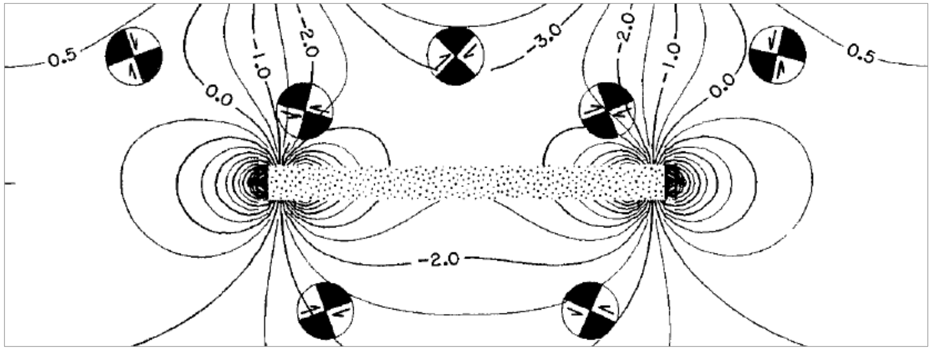
After several steps: $$u_i(x,y) = D \iint_\Omega g_i(x,y,\zeta,\xi)\, d\Omega,$$

$$\sigma_{ij}(x,y) = C \left[\iint_\Omega g_{ij}(x,y,\zeta,\xi)\, d\Omega - 2\pi\delta_\Omega \right]$$

where $D(\zeta,\xi) = \dfrac{(1-2\nu)\alpha p}{2\pi(1-\nu)G}$, $C = GD$

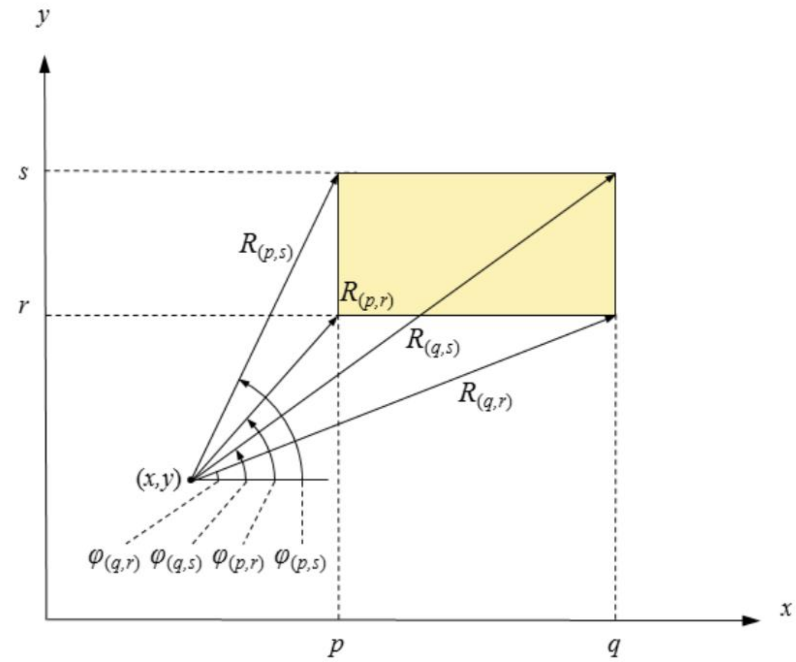
and g_i and g_{ij} are Green's functions for u_i and σ_{ij} .

Segall (1985, 1989, 1992), Segal & Fitzgerald (1989), Segal et al. (1994), Soltan-zadeh & Hawkes (2008), Marck et al. (2015)



Link to "nucleus of strain" concept: Rudnicki (2002)

Typical integral (for rectangle)



$$
\begin{aligned}
G_x(x,y) &= \int_p^q \int_r^s g_x(x,y,\zeta,\xi)\,d\xi\,d\zeta \\
&= \left\{\ln\left[(x-q)^2+(y-s)^2\right] - \ln\left[(x-p)^2+(y-s)^2\right]\right\} \times \frac{y-s}{4} \\
&\quad - \left\{\ln\left[(x-q)^2+(y-r)^2\right] - \ln\left[(x-p)^2+(y-r)^2\right]\right\} \times \frac{y-r}{4} \\
&\quad + \left\{\arctan\left[\frac{y-s}{x-q}\right] - \arctan\left[\frac{y-r}{x-q}\right]\right\} \times \frac{x-q}{2} \\
&\quad - \left\{\arctan\left[\frac{y-s}{x-p}\right] - \arctan\left[\frac{y-r}{x-p}\right]\right\} \times \frac{x-p}{2}
\end{aligned}
$$

With many thanks to Macsyma (open source symbolic manipulation tool)

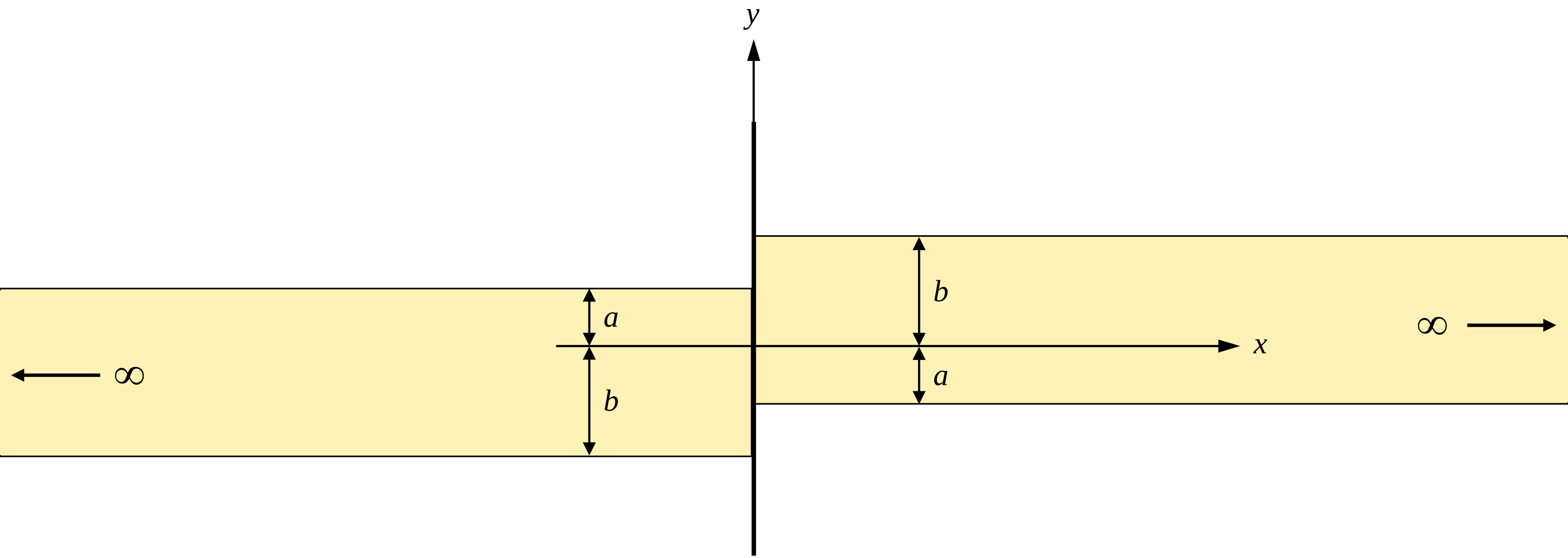
Results for faulted infinite reservoir

$$
\begin{aligned}
G_{xx}(x,y) &= \arctan 2\left[\frac{(a+b)x}{x^2+(y-b)(y+a)}\right] \\
&- \arctan 2\left[\frac{(a+b)x}{x^2+(y-a)(y+b)}\right],
\end{aligned}
$$

$$
\begin{aligned}
G_{yy}(x,y) &= \arctan 2\left[\frac{(y+a)}{-x}\right] - \arctan 2\left[\frac{(y-b)}{-x}\right] \\
&- \arctan 2\left[\frac{(y-a)}{x}\right] + \arctan 2\left[\frac{(y+b)}{x}\right],
\end{aligned}
$$

$$
G_{xy}(x,y) = \frac{1}{2}\ln\frac{\left[x^2+(y-a)^2\right]\left[x^2+(y+a)^2\right]}{\left[x^2+(y-b)^2\right]\left[x^2+(y+b)^2\right]}.
$$

Displaced (normal) fault model – even simpler



Reservoir width: infinite

a = 100 m

b = 200 m

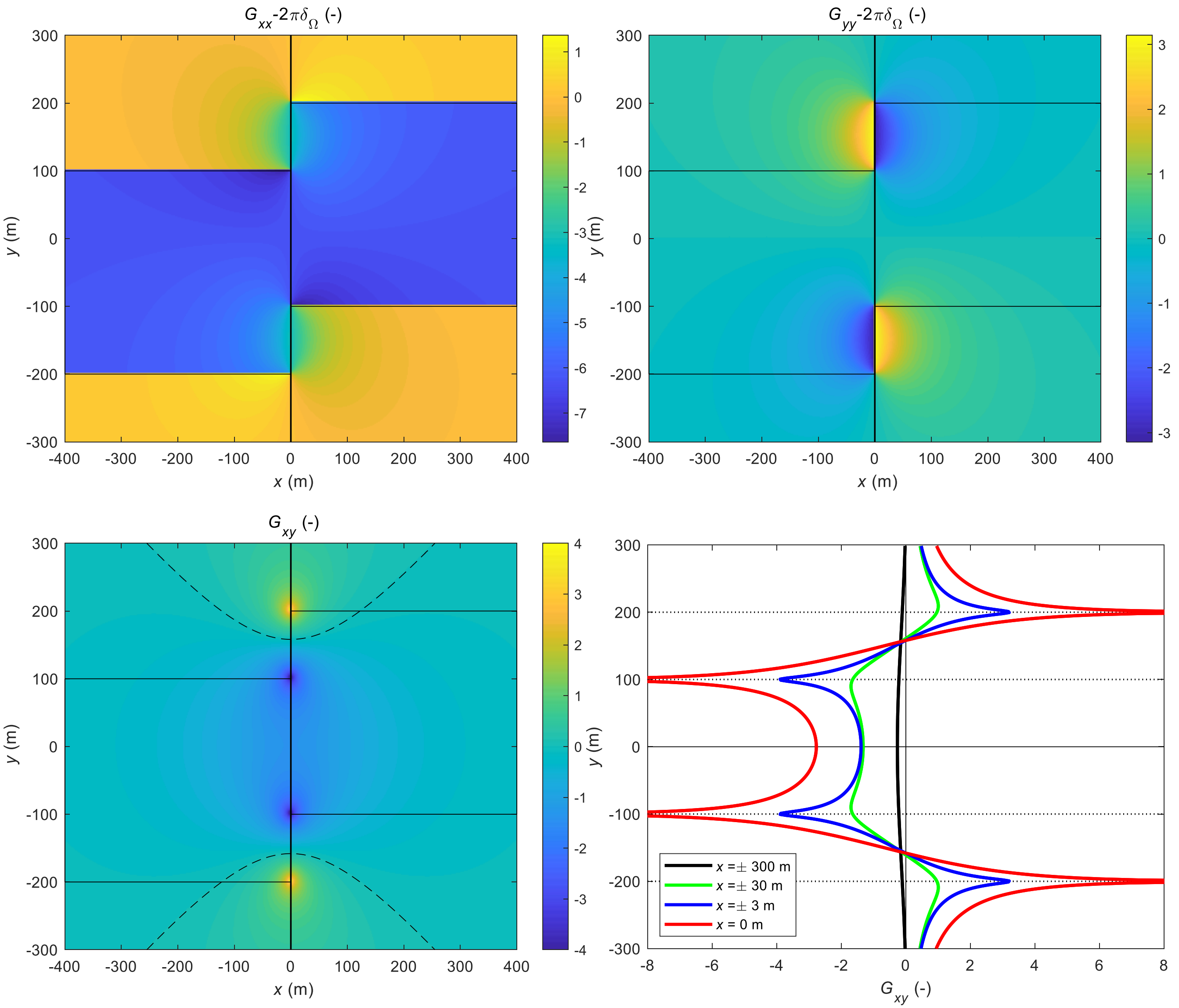
Reservoir height $h = a+b$ = 300 m

Fault throw $t = b - a$ = 100 m

Scaled stresses

$$\frac{\sigma_{xx}}{C} \qquad \frac{\sigma_{yy}}{C}$$

$$\frac{\sigma_{xy}}{C} \qquad \frac{\sigma_{xy}}{C}$$



Results for infinitely wide reservoir

$$\begin{aligned} G_{xx}(x,y) &= \arctan 2\left[\frac{(a+b)x}{x^2+(y-b)(y+a)}\right] \\ &- \arctan 2\left[\frac{(a+b)x}{x^2+(y-a)(y+b)}\right], \end{aligned}$$

$$\begin{aligned} G_{yy}(x,y) &= \arctan 2\left[\frac{(y+a)}{-x}\right] - \arctan 2\left[\frac{(y-b)}{-x}\right] \\ &- \arctan 2\left[\frac{(y-a)}{x}\right] + \arctan 2\left[\frac{(y+b)}{x}\right], \end{aligned}$$

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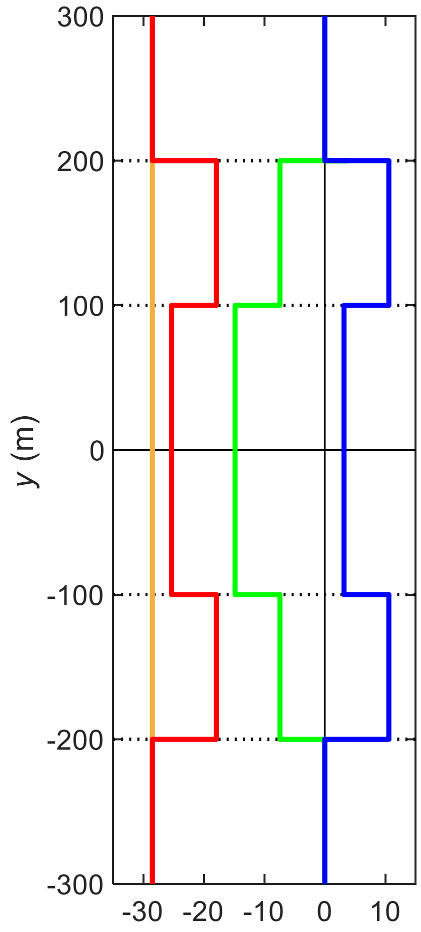
$$
G_{xy}(x,y) = \frac{1}{2}\ln\frac{[x^2+(y-a)^2][x^2+(y+a)^2]}{[x^2+(y-b)^2][x^2+(y+b)^2]}.
$$

Singularities at $y = \pm a$ and $y = \pm b$ (top and bottom of left/right reservoirs)

Stresses and slip boundaries



a) Horizontal stresses (MPa)

orange: σ'^{0}_{xx}, *green:* σ_{xx},

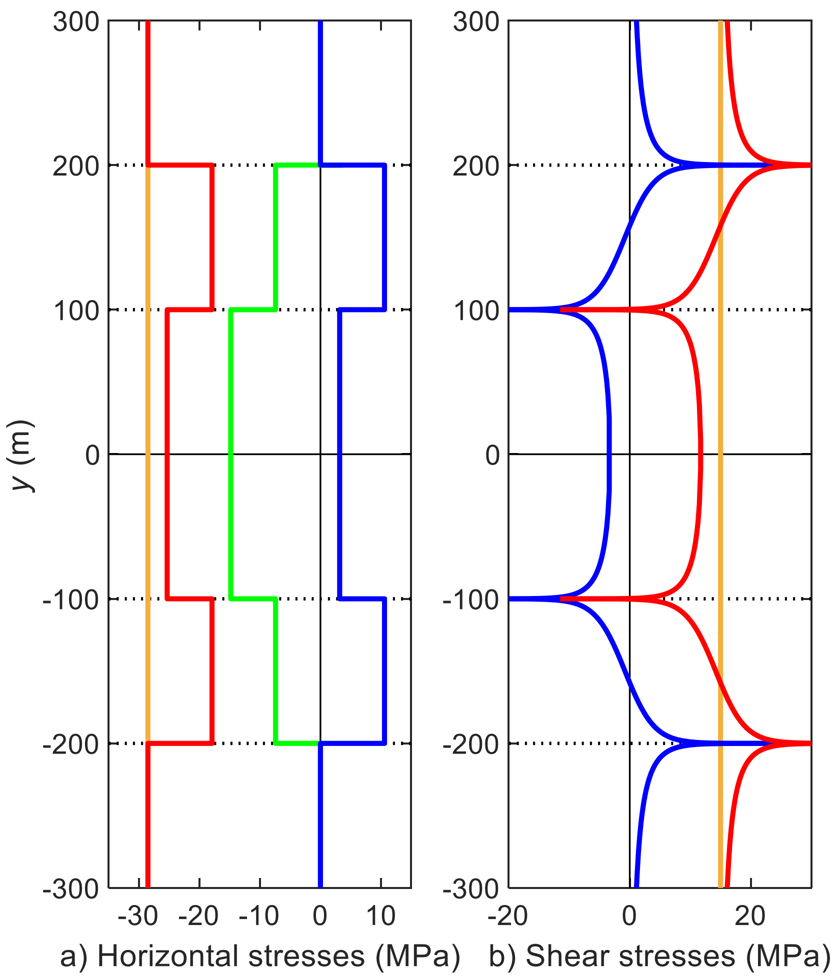
blue: σ'_{xx}, *red:* $\sigma'^{0}_{xx} + \sigma'_{xx}$

p^{0} = 35 MPa

p = 20 MPa

p^{tot} = 55 MPa

Stresses and slip boundaries



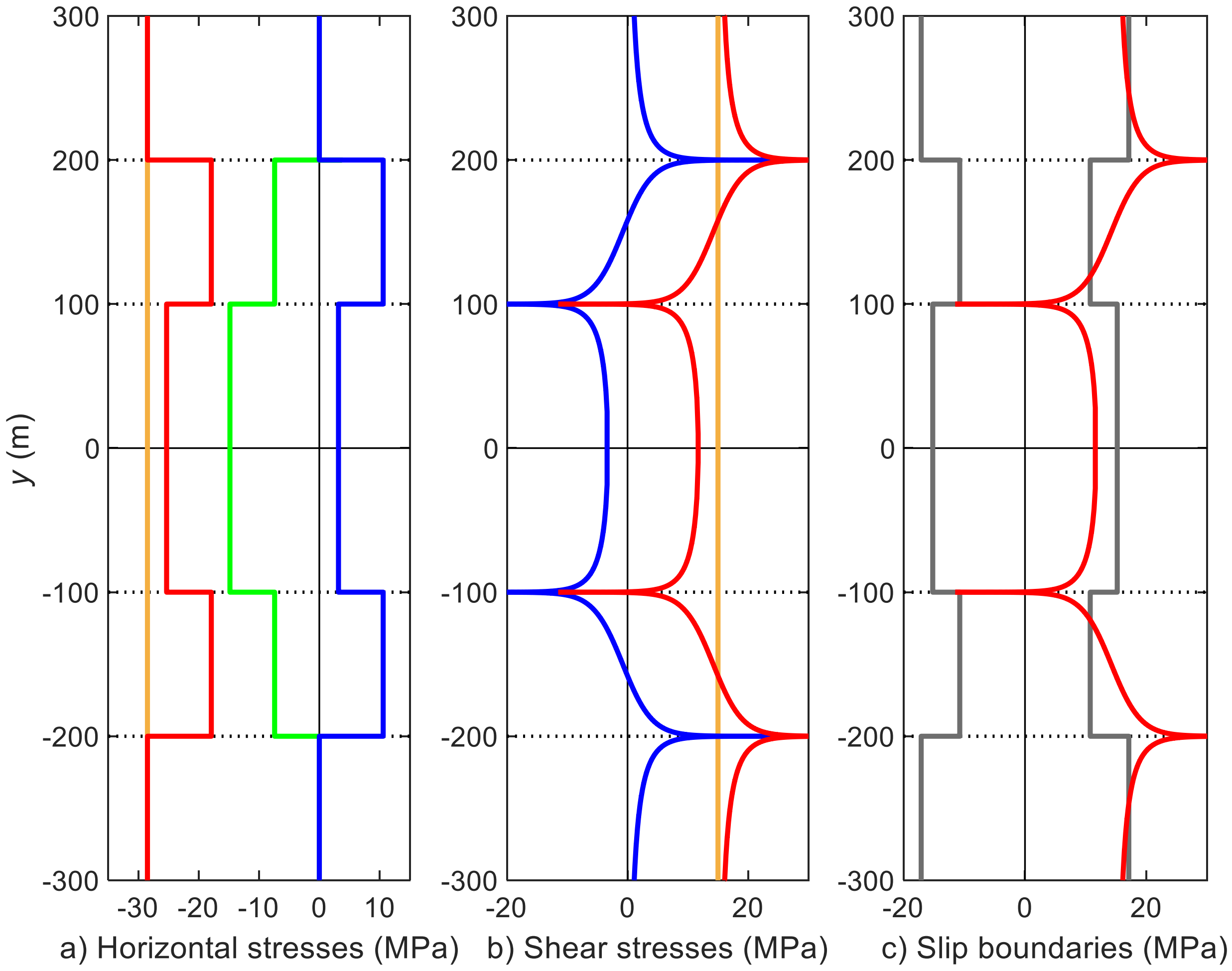
orange: σ_{xy}^0

blue: σ_{xy}

red: $\sigma_{xy}^0 + \sigma_{xy}$

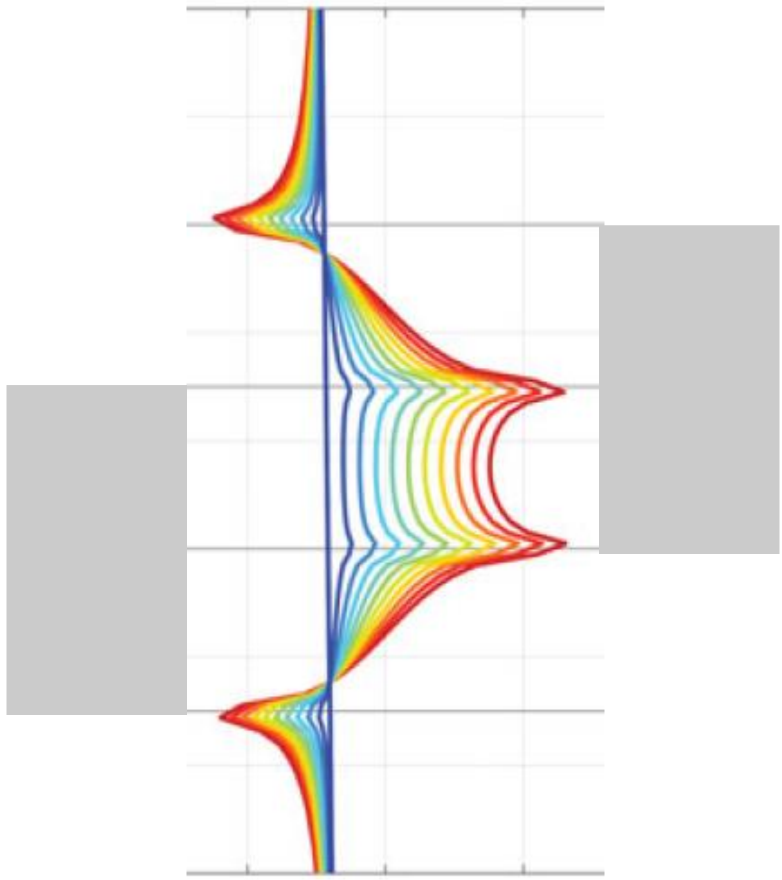
p = 20 MPa

Stresses and slip boundaries

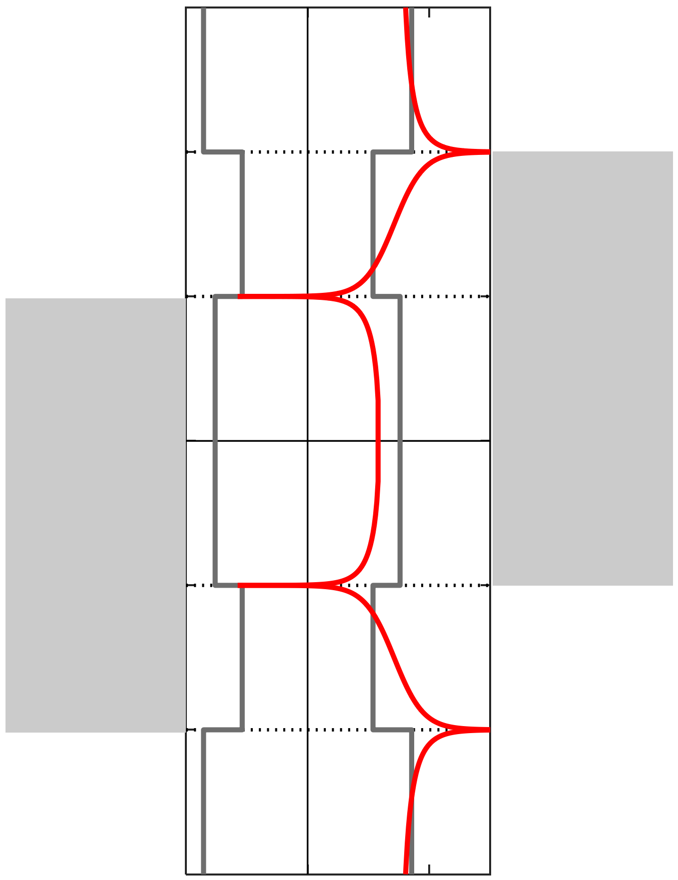


p = 20 MPa

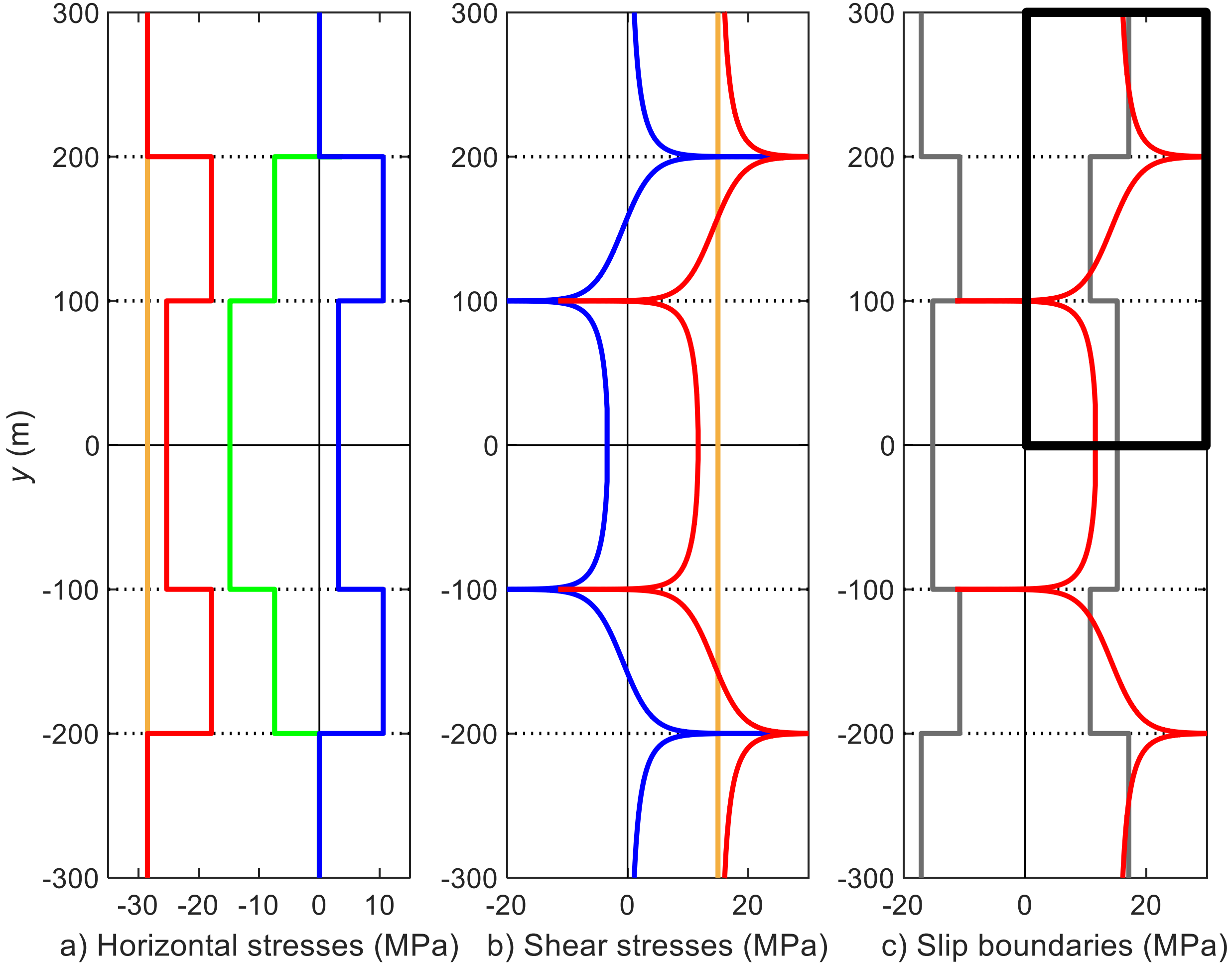
What's wrong?



Van Wees et al. 2017



Stresses and slip boundaries

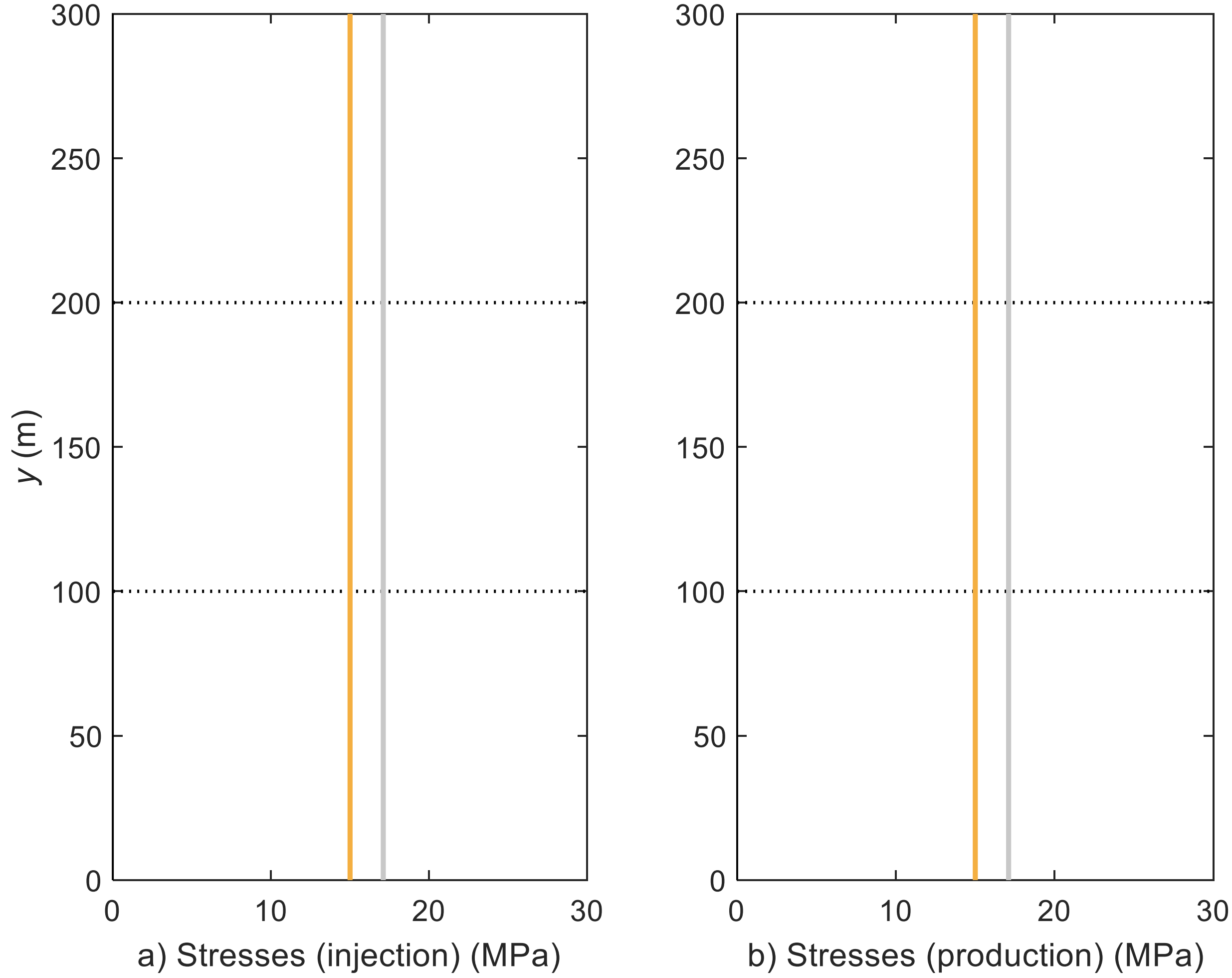


p = 20 MPa

Injection and production

SCU = 0.88 @ μ_{st} = 0.6
SCU = 1.05 @ μ_{dyn} = 0.5

p = 0 MPa

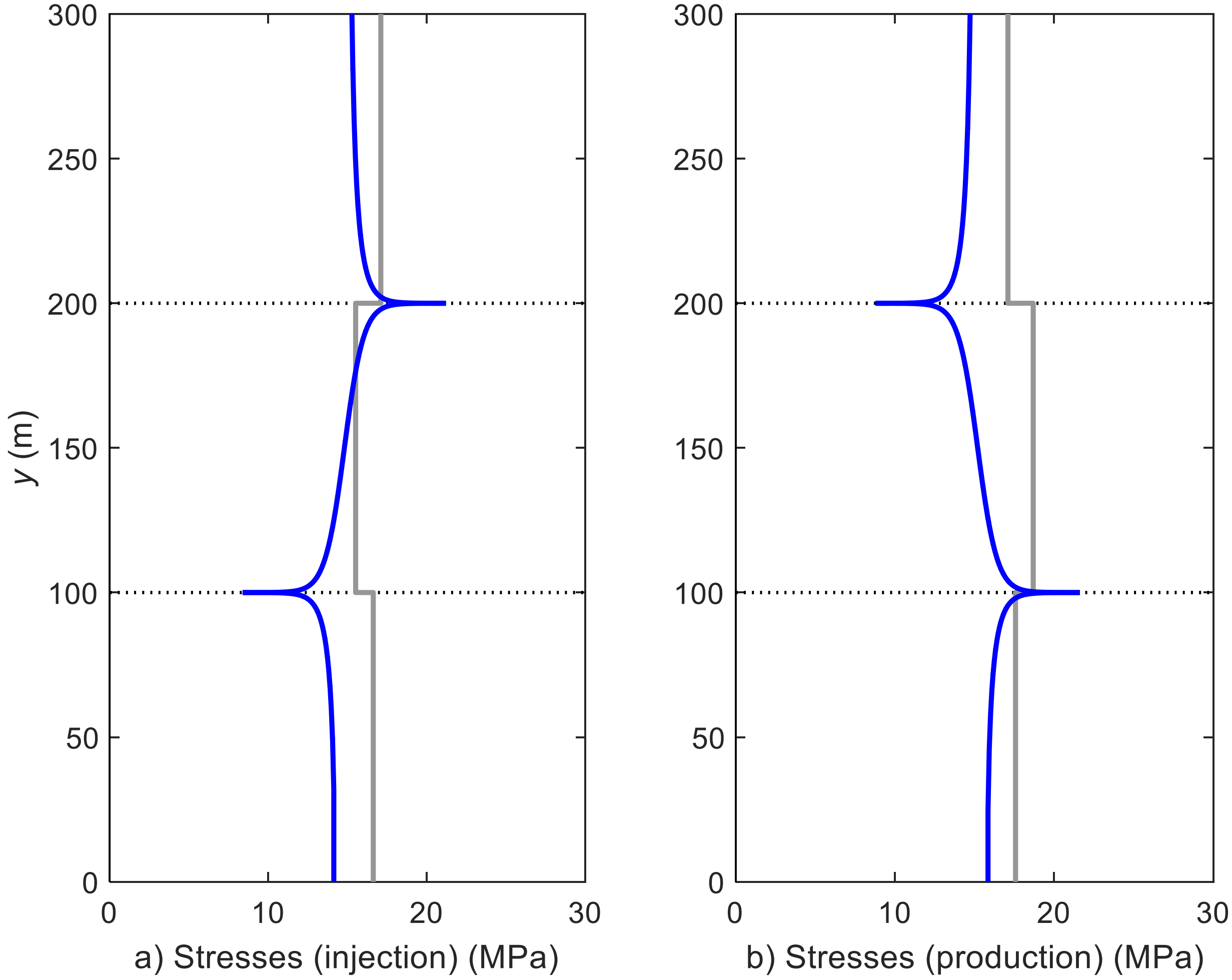


p = 0 MPa

Injection and production

p = 5 MPa

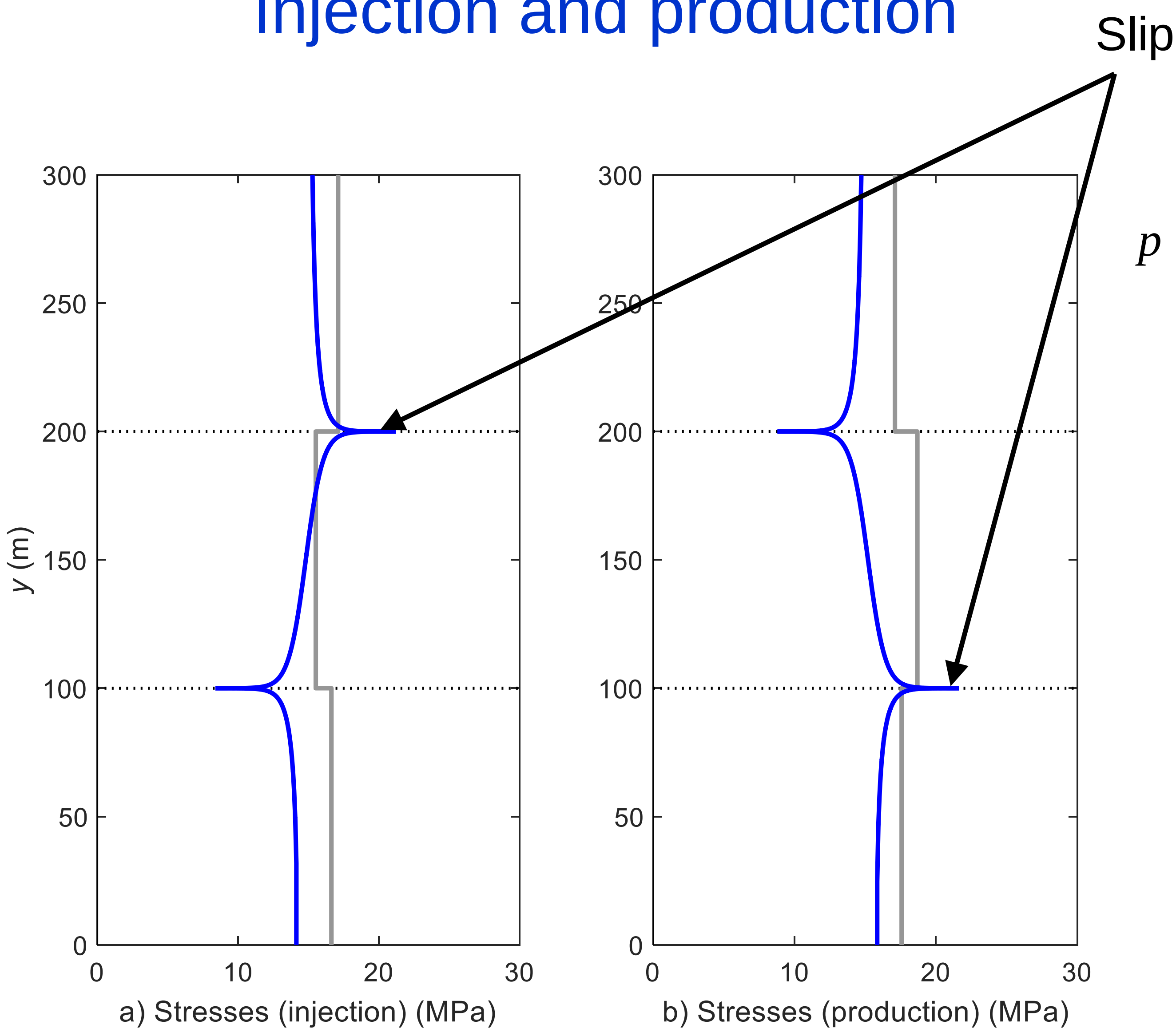
p = -5 MPa



Injection and production

p = 5 MPa

p = -5 MPa



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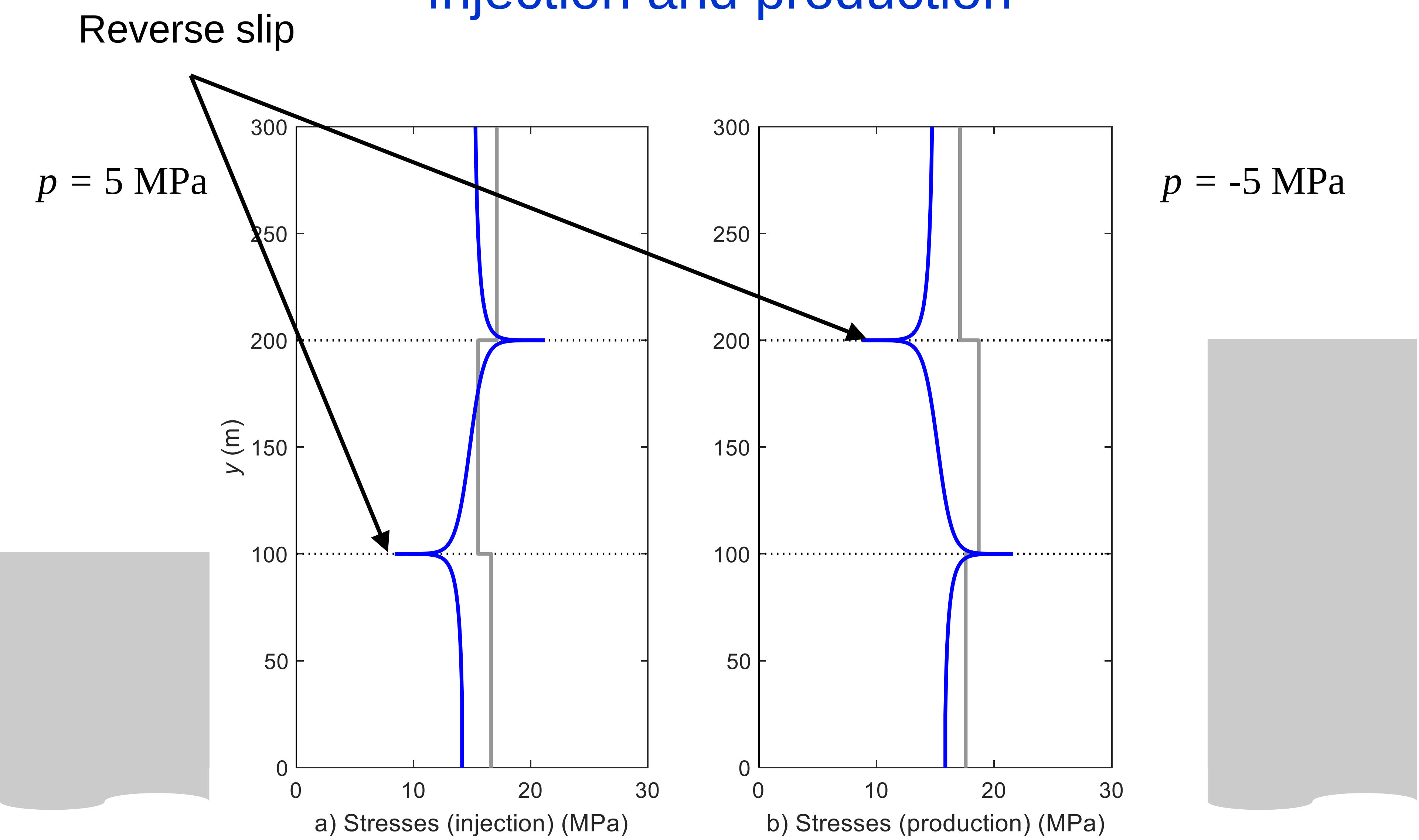
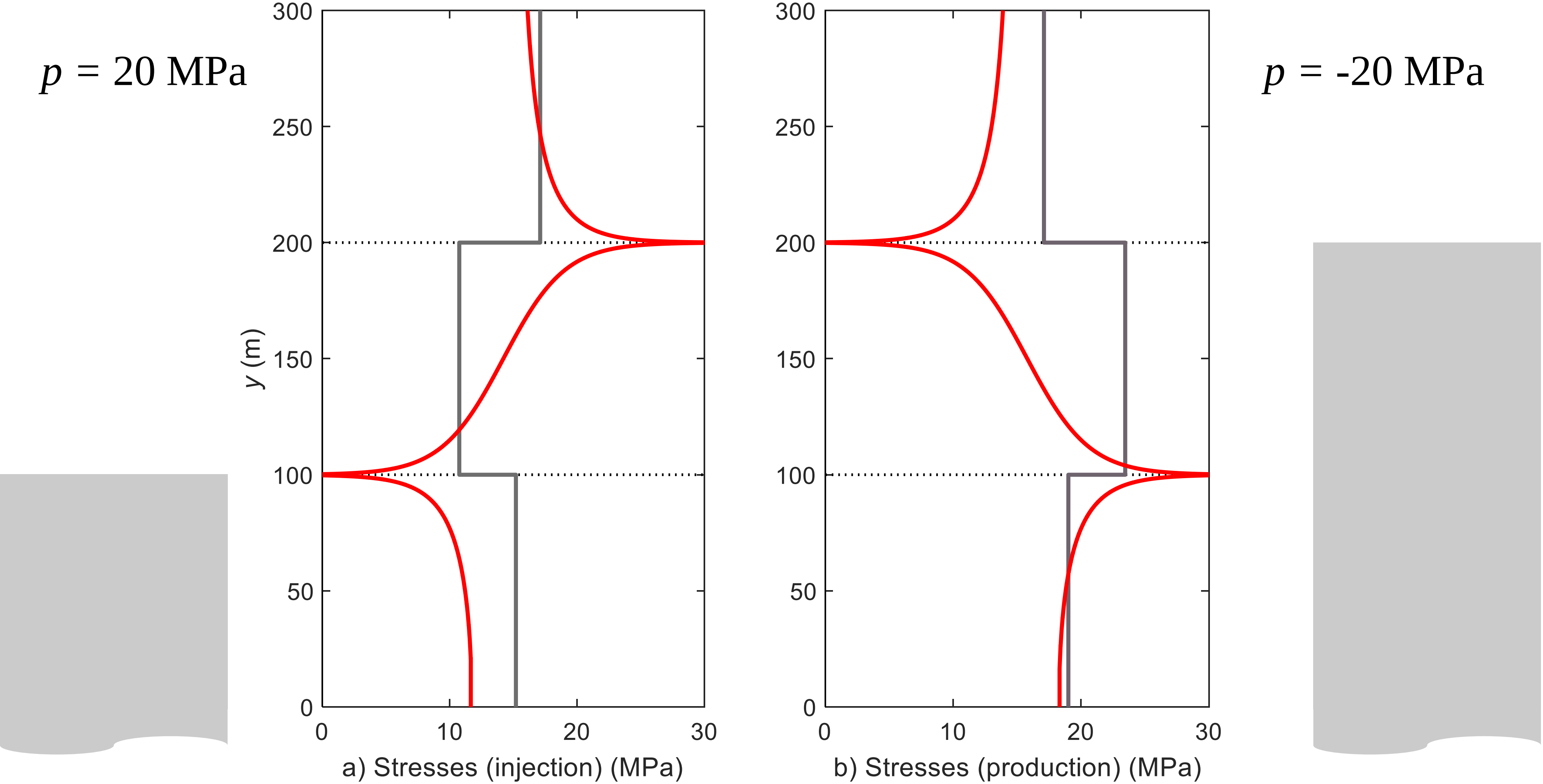


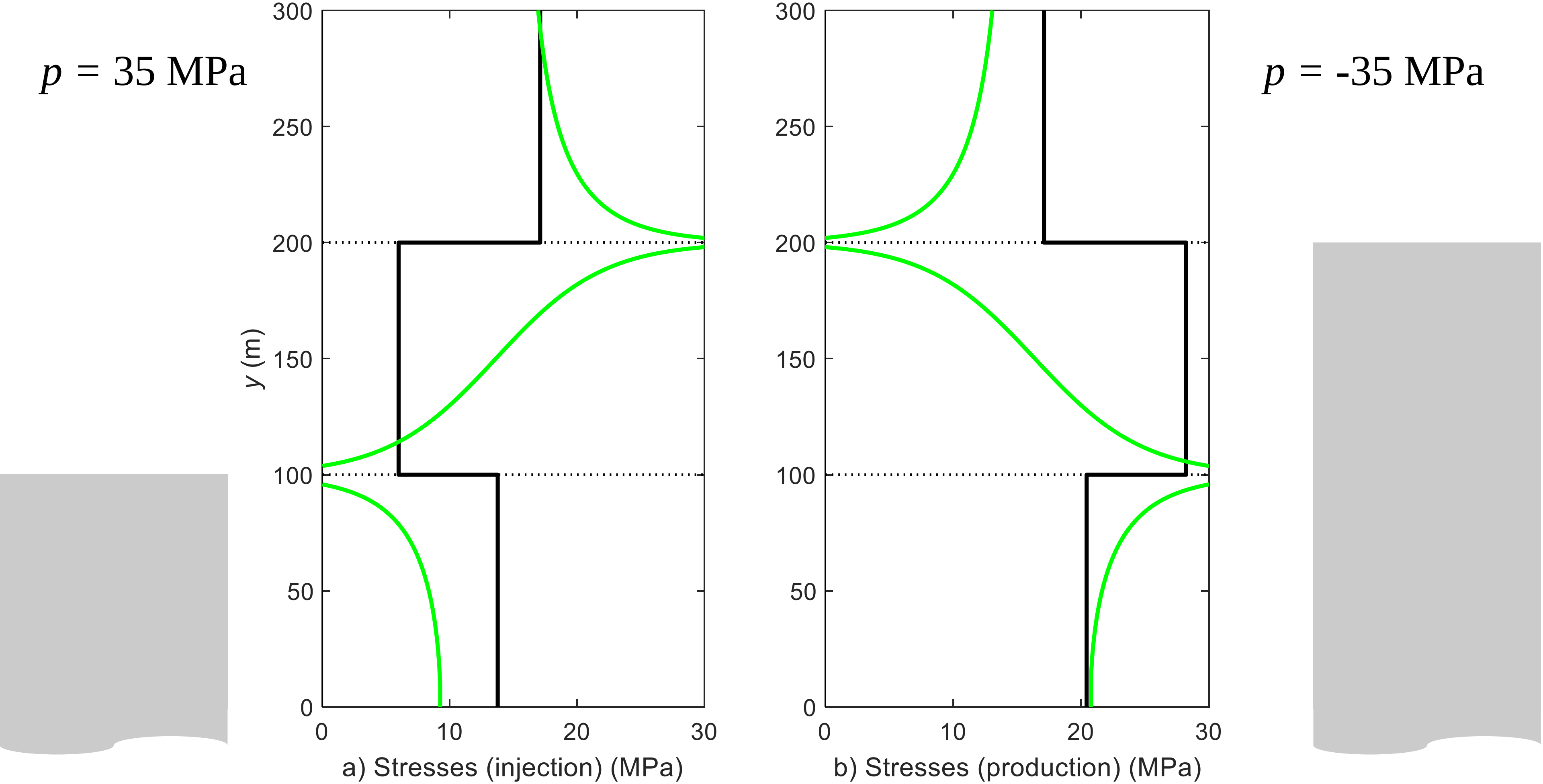
Injection and production



Injection and production

p = 35 MPa

p = -35 MPa



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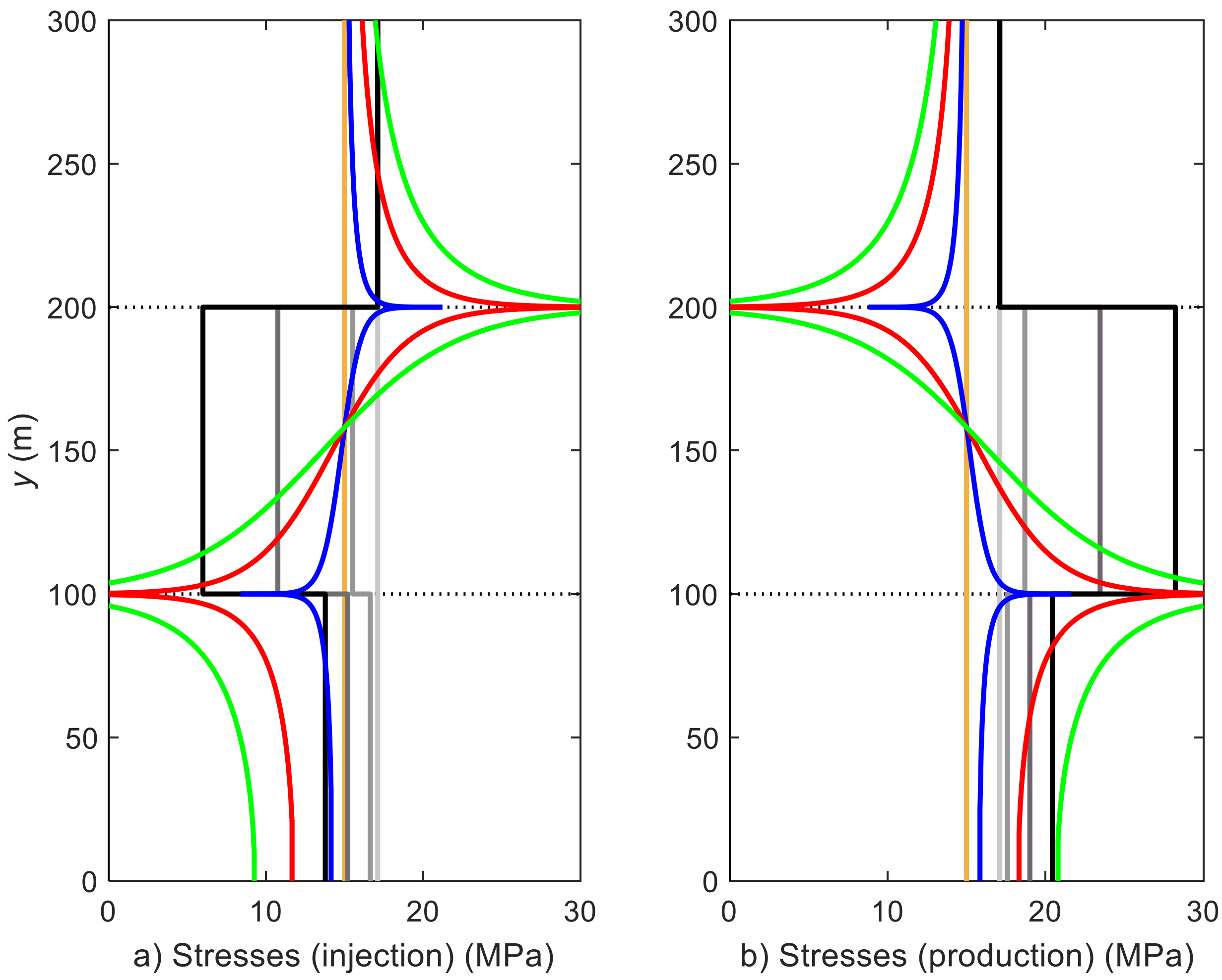
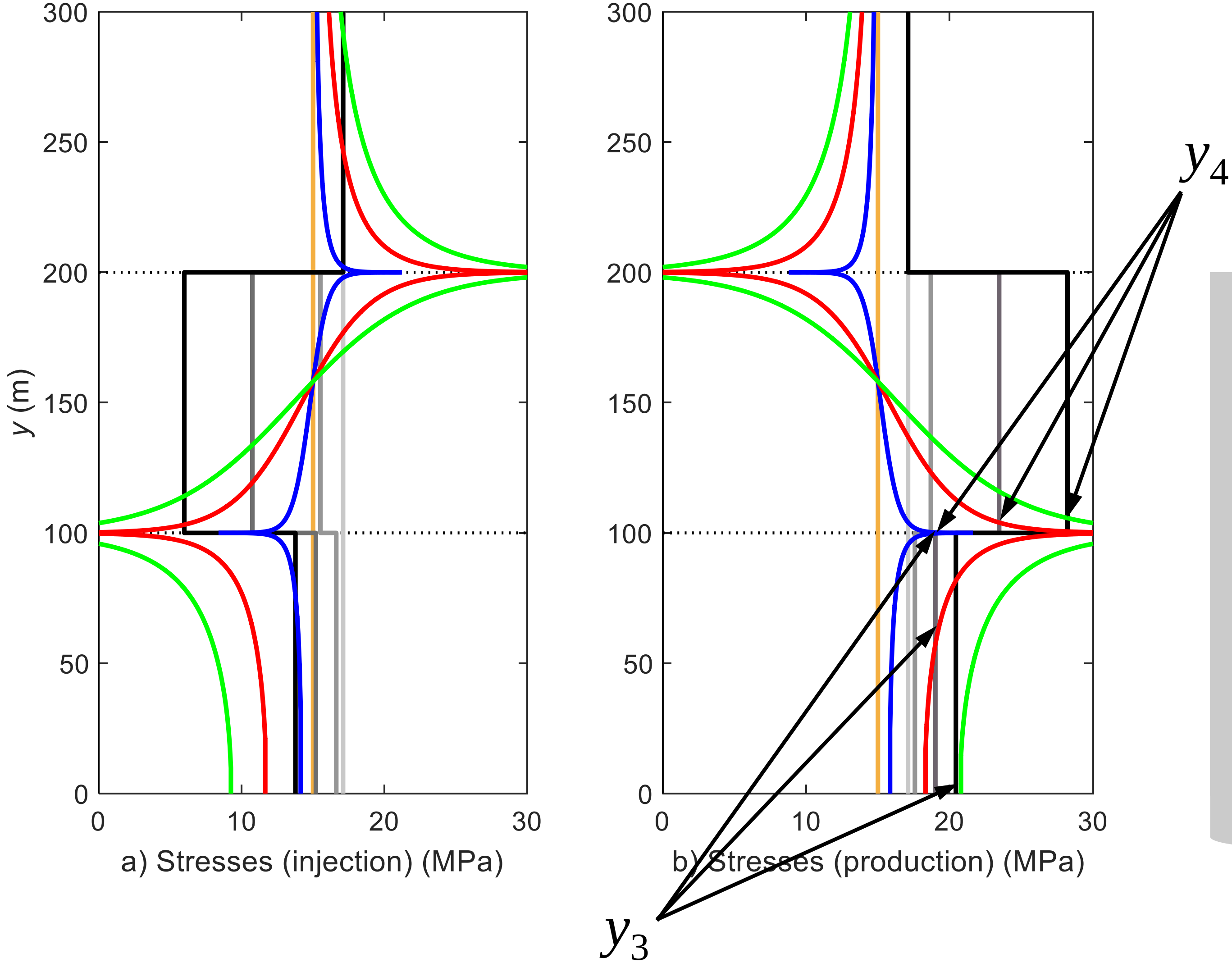
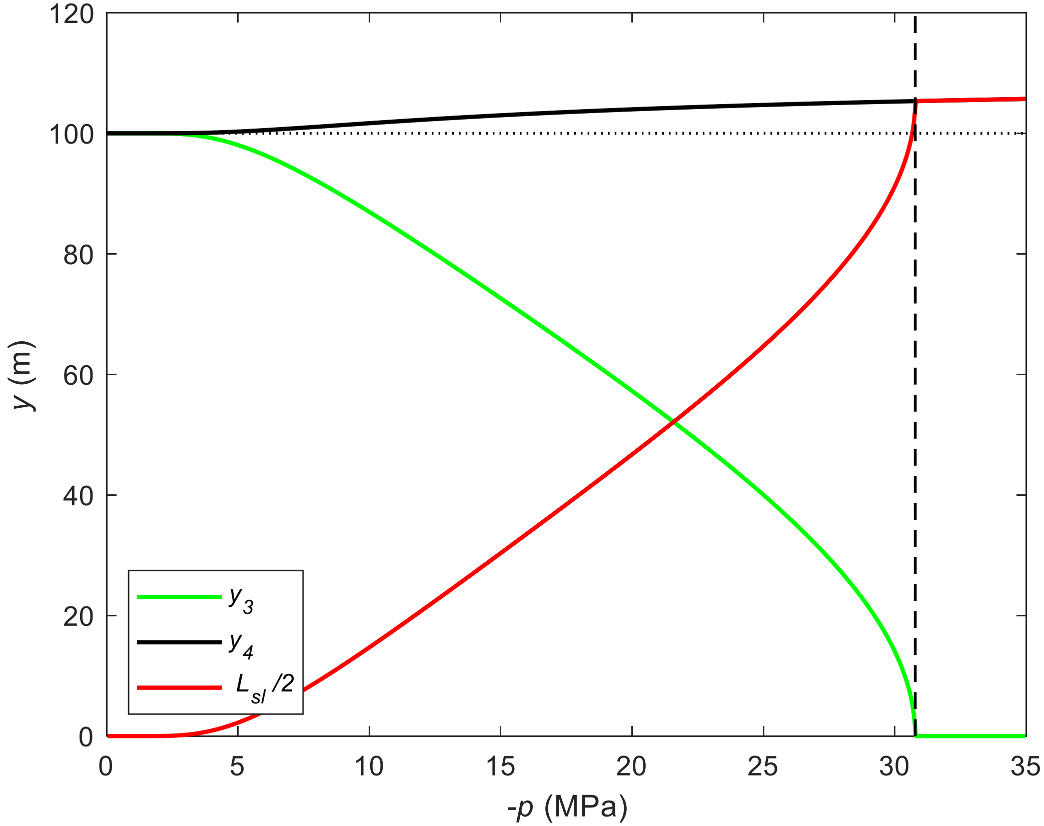


Injection and production



Intersections and slip patch half-length $L_{sl}/2$



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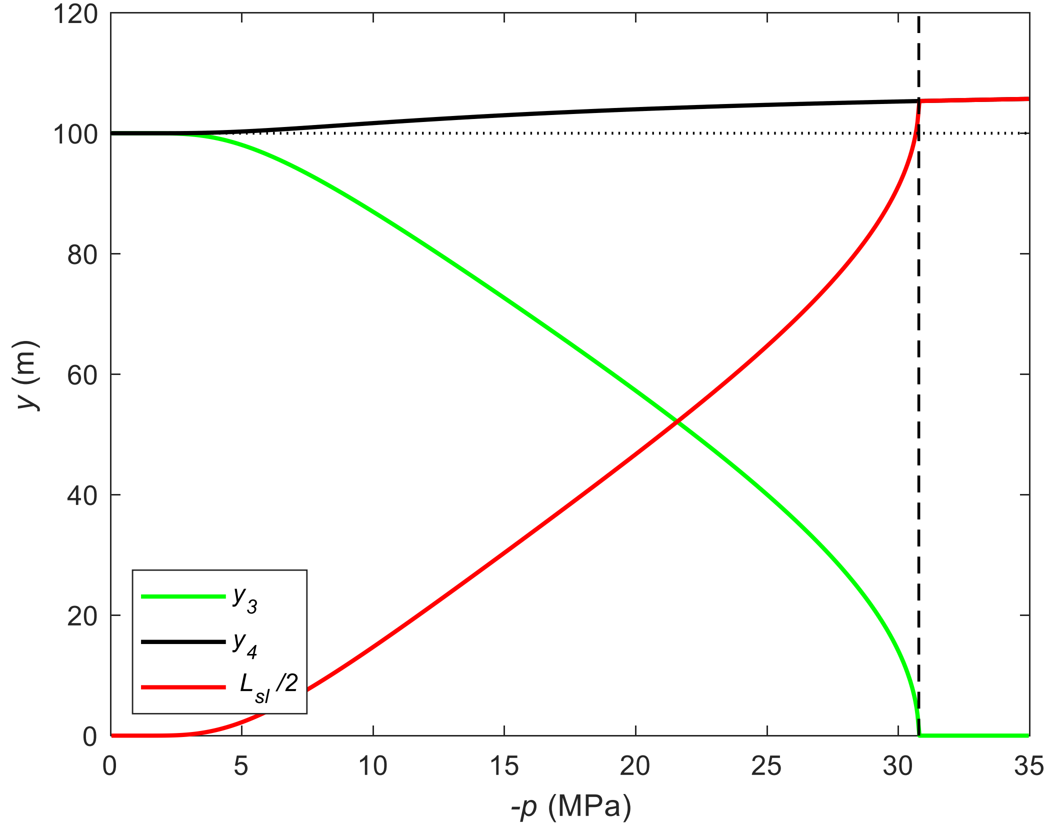
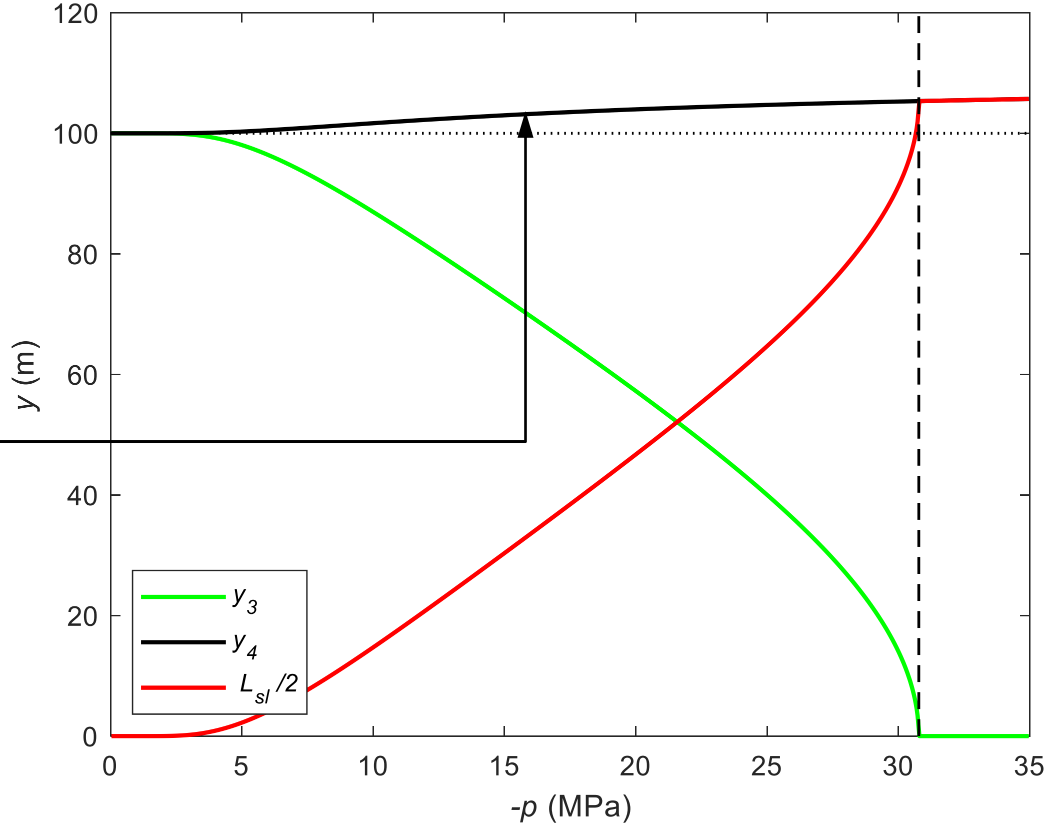


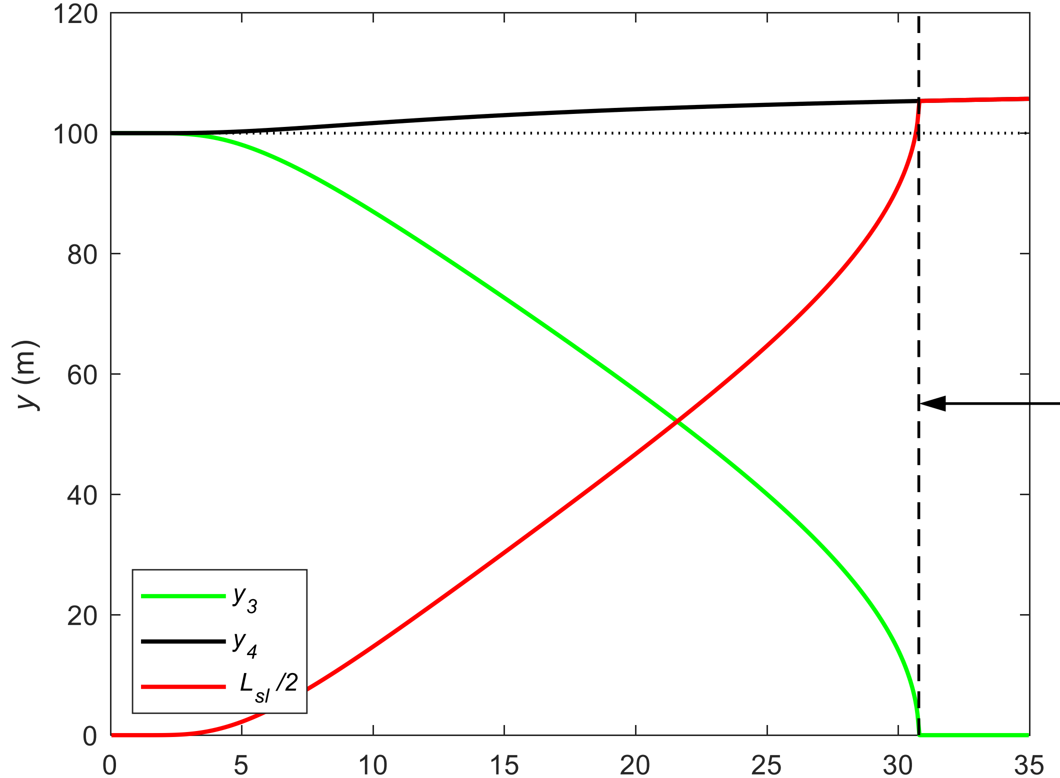
Same result as derived by Van den Bogert (2015)

Intersections and slip patch half-length $L_{sl}/2$



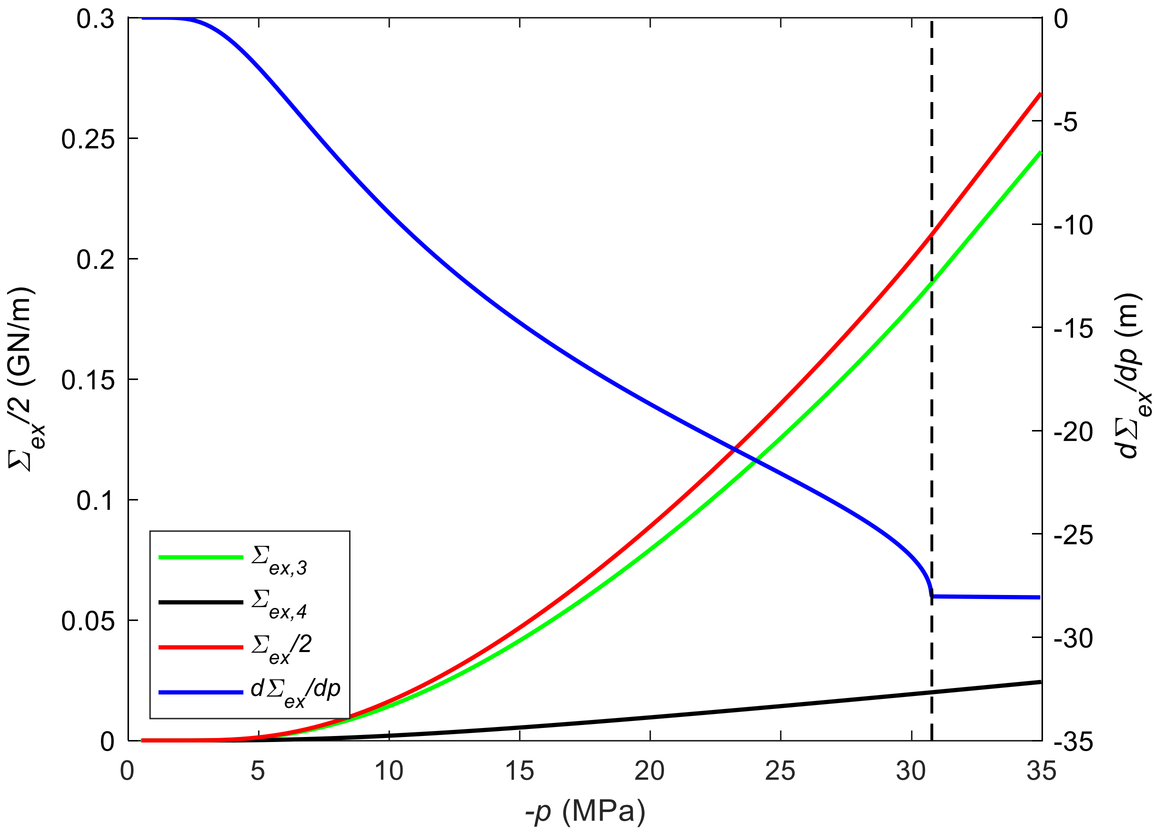
$$y_{1,4} = \mp\sqrt{\frac{\gamma\, b^2 - \sqrt{\gamma}\,\left(b^2 - a^2\right) - a^2}{\gamma - 1}}, \;\; \gamma = exp\left[\frac{\sigma_{sl}(y_4) - \sigma^0_{xy}}{C/2}\right]$$

Intersections and slip patch half-length $L_{sl}/2$



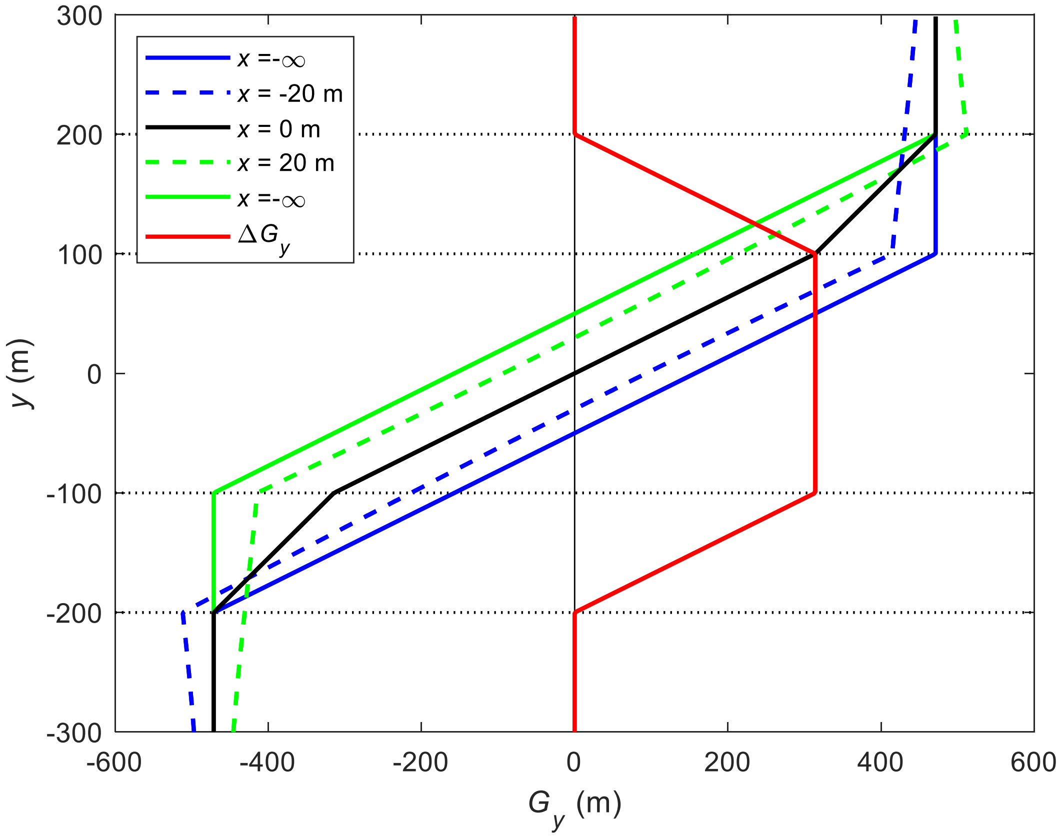
$$p_{mer} = -\frac{\pi}{\alpha}\left[\frac{\dfrac{1-\nu}{1-2\nu}\left(\sigma^{0}_{xy}+\mu_{st}\sigma'^{0}_{xx}\right)}{2\ln\dfrac{a}{b}+\pi\mu_{st}\left(\dfrac{1-\nu}{1-2\nu}-1\right)}\right]$$

Excess shear force in fault ("shear glut")



$$\Sigma_i^{ex} = \frac{C}{2}\left\{ a \ln\left[\frac{(2a)^4 (y_i - a)^2}{(a-b)^2 (a+b)^2 (y_i + a)^2}\right] + b \ln\left[\frac{(a-b)^2 (y_i + b)^2}{(a+b)^2 (y_i - b)^2}\right] + y_i \ln\left[\frac{(y_i - b)^2 (y_i + b)^2}{(y_i - a)^2 (y_i + a)^2}\right]\right\} + (a - y_i)\left[\sigma_{xy}^0 - \sigma_{sl}(y_i)\right]$$

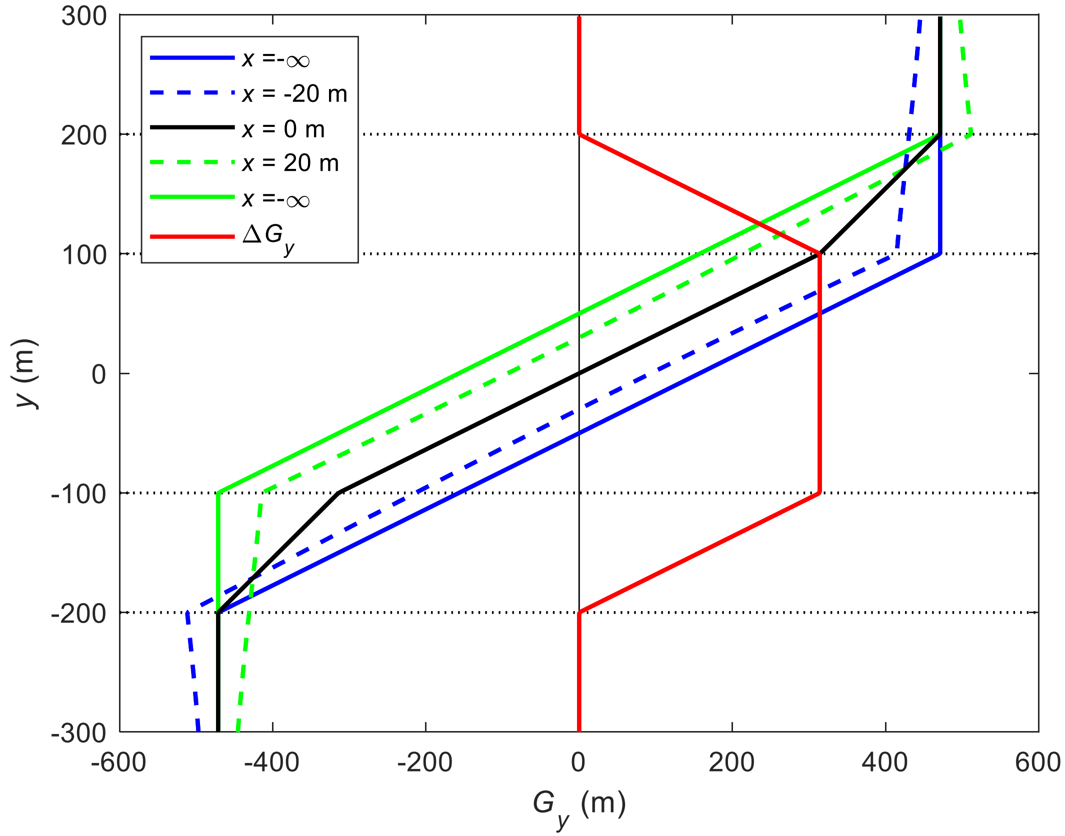
Scaled vertical displacements



$$G_y = \frac{u_y}{D}$$

$$\begin{aligned} G_y(-\infty, y) &= \frac{\pi}{2}\left(|y+b| - |y-a|\right), \\ G_y(\infty, y) &= \frac{\pi}{2}\left(|y+a| - |y-b|\right). \end{aligned}$$

Seismic moment



$$M = GD\int_{-b}^{b} \Delta G_y \, dy = C\pi(b^2 - a^2) = C\pi ht = \frac{(1-2\nu)\alpha p}{2(1-\nu)} ht,$$

Same result as derived by Bourne and Oates (2017)

Conclusions

Results from various earlier studies confirmed and sometimes sharpened, in particular from (Van den Bogert, 2015; Buijze et al., 2017; Van Wees et al., 2017; Zbinden et al. 2017; Buijze et al., 2019):

- Elastic stress patterns independent of *p;* only dependent on geometric reservoir parameters.
- Production results in peaked shear stress profile: slip-provoking at internal corners; slip arresting at external corners.
- Two slip patches in the production case grow mainly inwards at an exceedingly high rate with decreasing reservoir pressure.

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Results from many earlier studies confirmed and sometimes sharpened, in particular from (Van den Bogert, 2015; Buijze et al., 2017; Van Wees et al., 2017; Zbinden et al. 2017; Buijze et al., 2019):

- Elastic stress patterns independent of *p;* only dependent on geometric reservoir parameters.
- Production produces peaked shear stress profile: slip-provoking at internal corners; slip arresting at external corners.
- Two slip patches in the production case grow mainly inwards at an exceedingly high rate with decreasing reservoir pressure.

New insights from the present study (at least to me ☺):

- Infinitely large shear stresses in a displaced fault => smallest amount of injection or production will result in slip; no such thing as a slip-free maximum injection or minimum production pressure.
- Marked difference between the shear stress patterns for injection and production: for injection, slip grows always into over/underburden; for production, slip could well stay in the reservoir.