The Static Behaviour of Induced Seismicity

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Schatzalp Workshop
Understanding and Modeling of Induced Seismicity (II)
Thursday, 16 March 2017, 08:35-08:50
The 2006 Basel EGS textbook example

2006 Basel EGS data sources:
Häring et al. (2008);
Kraft & Deichmann (2014)
The 2006 Basel EGS textbook example

\[
\begin{align*}
\mu(t) &= 10^{a_f} 10^{-10^{b_M} \Delta V(t)} ; t \leq t_{\text{shut-in}} \\
\mu(t) &= \mu(t_{\text{shut-in}}) \exp \left(-\frac{t - t_{\text{shut-in}}}{\tau}\right) ; t > t_{\text{shut-in}}
\end{align*}
\]

2006 Basel EGS data sources:
Häring et al. (2008); Kraft & Deichmann (2014)
Very reasonable results obtained quite systematically with very SIMPLE MODEL

\[
\begin{cases} 
\mu(t) = 10^{afb} 10^{-bM_c} \Delta V(t) & ; t \leq t_{\text{shut-in}} \\
\mu(t) = \mu(t_{\text{shut-in}}) \exp \left( -\frac{t - t_{\text{shut-in}}}{\tau} \right) & ; t > t_{\text{shut-in}} 
\end{cases}
\]

Linear relationship $\mu \approx \Delta V$
✓ Empirical
✓ $a_{fb}$ equivalent to seismogenic index $\Sigma$ (Shapiro et al.)

Normal diffusion
✓ 5 out of 7 time series best described by exponential function (stretched exp. better in 2 cases)
✓ Same principle as for tectonic aftershocks (Mignan, GRL 2015)

Source: Mignan et al. (sub.)
This model can be based on simple physics, using GEOMETRY instead of poroelasticity.

**Poroelastic approach**

Fluid injection overpressure ➔ Fluid flow in porous medium ➔ Induced seismicity

*Biot’s theory*

**Geometrical approach**

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*N-C PAST postulate*

**Description length**

*Source: Mignan (NPG 2016)*
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**Geometrical approach**

- Fluid injection overpressure
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- Induced seismicity

- Algebraic model (**INTEGRABLE** in analytical risk management, e.g., closed-form TLS)
- Few physical parameters (**PARSIMONIOUS**)
- Same physics as tectonic earthquakes (**UNIFYING**)

**Description length**

*Source: Mignan (NPG 2016)*
REDUCTIONIST GEOMETRIC approach with STATIC stress top-down loading as driver

- **Opposite to complexity theory**, which is holistic (stem of “complex” means “intertwined”), dynamic, controlled by bottom-up triggering (& critical points)

- **Postulate.** Seismicity is strictly categorized into three regimes of constant spatiotemporal densities – background $\delta_0$, quiescence $\delta_-$ and activation $\delta_+$ (with $\delta_- < \delta_0 < \delta_+$) – and depends on the static stress step function $\delta(\sigma)$ with $\Delta\sigma$ the background static stress amplitude range

  $$\delta(\sigma) = \begin{cases} 
  \delta_- & \text{if } \sigma < -\Delta\sigma_* \\
  \delta_0 & \text{if } \sigma \leq |\Delta\sigma_*| \\
  \delta_+ & \text{if } \sigma > \Delta\sigma_* 
\end{cases}$$

- **Building of “seismicity solids”:**
  - Permanent static stress field
    $$\sigma(r,t) \propto \frac{\Sigma(t)}{r^3}$$
  - Seismicity solid envelope
    $$r_*(t) \propto \left(\frac{\Sigma(t)}{\Delta\sigma_*}\right)^{\frac{1}{3}}$$
  - Seismicity rate function
    $$\mu(t) \propto \delta kr_*^d(t)$$
Originally coined **NON-CRITICAL** precursory accelerating seismicity theory (N-C PAST)

Simulations of precursory seismicity from algebraic model

\[ \mu(t) \propto \delta w \pi \left( \frac{t}{\Delta \sigma_*} \right)^{\frac{2}{3}} \]

Observations (2009 L'Aquila mainshock)

Source: Mignan (GRL 2012)
“N-C PAST Postulate” also explains parabolic spatial front & linear relationship $\mu \approx \Delta V$

1. Parabolic front of induced seismicity = **Activation solid** driven by borehole overpressure

\[
    r_*(t) \propto \left( \frac{K\Delta V(t)}{\Delta \sigma_*} \right)^{\frac{1}{3}}
\]

Source: Mignan (NPG 2016)

2. Linear relationship between induced seismicity rate & flow rate = direct consequence of 1

\[
    \mu(t) \propto \delta_+ \frac{4\pi}{3} \frac{K}{\Delta \sigma_*} \Delta V(t)
\]
More complicated cases (stem of “COMPLICATED” meaning “FOLDED”)

1. Sum of two pressure fields, e.g. overpressure + underpressure in production phase
More complicated cases (stem of “complicated” meaning “folded”)

1. **Sum of two pressure fields**, e.g. overpressure + underpressure in production phase

2. **Sum of overpressure field + remnant of permanent static stress field** of an active fault

*Source: Mignan (NPG 2016)*

*Source: Shapiro et al. (GRL 2006), KTB 2004/5 anisotropy*
Note on Aftershocks & post-injection relaxation

✓ **Omori law (power law) ill-defined**: $c > 0$ infers that singularity occurs before mainshock (*Kagan & Houston* 2005)

✓ **A stretched exponential function** fits aftershocks better than a standard power-law (*Mignan*, GRL 2015); similar for post-injection cases (against complexity?)

✓ **Subdiffusion** explainable by **STATIC trap model** (*Grassberger & Procaccia* 1982) with stretching explained by **TOPOLOGY** of traps (fractal fault network)

<table>
<thead>
<tr>
<th>Name</th>
<th>Distribution $p(x) = Cf(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power law</td>
<td>$x^{-\alpha}$ $(\alpha - 1)x_{\text{min}}^{\alpha - 1}$</td>
</tr>
<tr>
<td><strong>Power law with cutoff</strong></td>
<td>$x^{-\alpha}e^{-\lambda x}$ $\frac{\lambda^{1-\alpha}}{\Gamma(1-\alpha, \lambda x_{\text{min}})}$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$e^{-\lambda x}$ $\lambda e^{\lambda x_{\text{min}}}$</td>
</tr>
<tr>
<td>Stretched exponential</td>
<td>$x^{\beta - 1} e^{-\lambda x^\beta}$ $\beta \lambda e^{\lambda x_{\text{min}}^\beta}$</td>
</tr>
<tr>
<td>Log-normal</td>
<td>$\frac{1}{x} \exp \left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right] \sqrt{\frac{2}{\pi\sigma^2}} \text{erfc} \left(\frac{\ln x_{\text{min}} - \mu}{\sqrt{2\sigma}}\right)^{-1}$</td>
</tr>
</tbody>
</table>

Source: *Clauset et al.* (2009)
Induced seismicity references:

See also on geometric origin of seismicity:

More on physics:
Closed-form TLS (risk mitigation)

Game theory (decision making under uncertainty)
AGAINST Complexity Theory (stem of complex meaning “intertwined”)

✓ Holistic – Bottom-up triggering – Dynamic – Critical

Source: Mignan (Tectonophysics 2011)
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✓ Self-Organized Criticality (SOC) gives power-law freq.-size distr.

Bak-Tang-Wiesenfeld model

Analogue of Gutenberg-Richter law?

Source: Mignan (Tectonophysics 2011)

Local interactions lead to system behaviour
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- Movable Cellular Automata mimic rock lab experiments
  - CA where laws of physics are implemented (e.g., Hooke’s law, friction’s laws)
  - Extrapolating lab results to crust behaviour makes sense in Complexity paradigm (bottom-up process, scale-invariant)

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✓ In terms of GEOMETRY: can we really extrapolate results from a confined cylindrical rock sample to a spherical layer with free surface (crust)? Different TOPOLOGIES