Schweizerischer Erdbebendienst
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Why ML and MW for small earthquakes scale as 1.5:1 instead of 1:1

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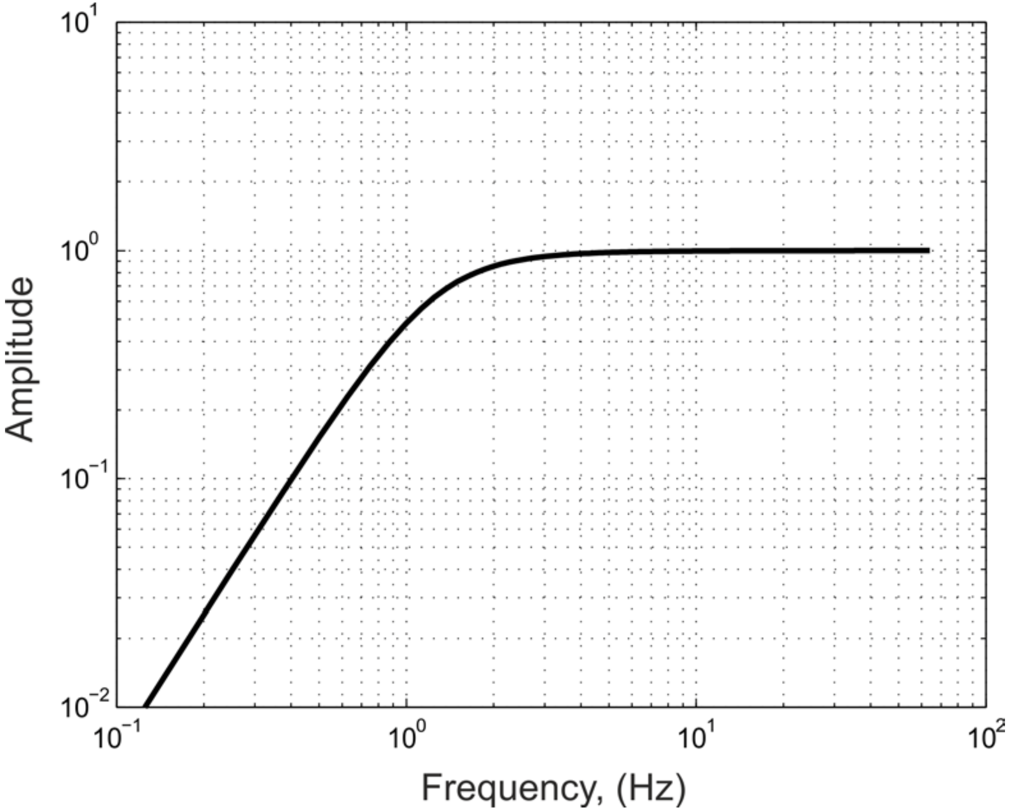
Outline

- In theory ML and MW should scale 1:1
- In reality they scale about 1.5:1
- Simulations with Q confirm the 1.5:1 scaling
- Theory with Q shows why
- The added effect of the W-A response
- Consequences for Gutenberg-Richter

Basic principles:

Richter: $M_L = \log A_{WA} - \log A_0$



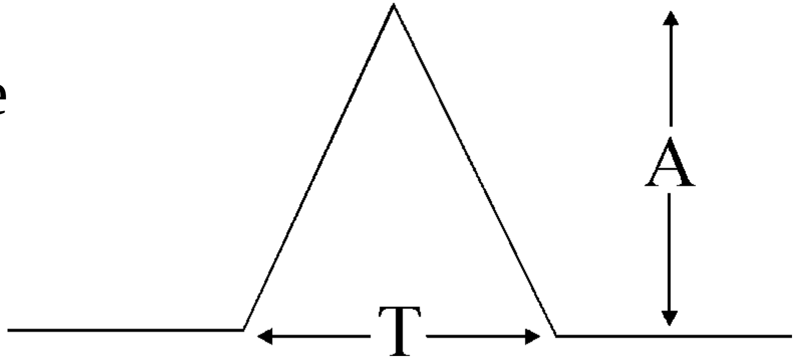
Frequency response of the Wood-Anderson seismometer

Basic principles:

Richter: $M_L = \log A_{WA} - \log A_0$

(1) $M_L = \log A - \log A_0$

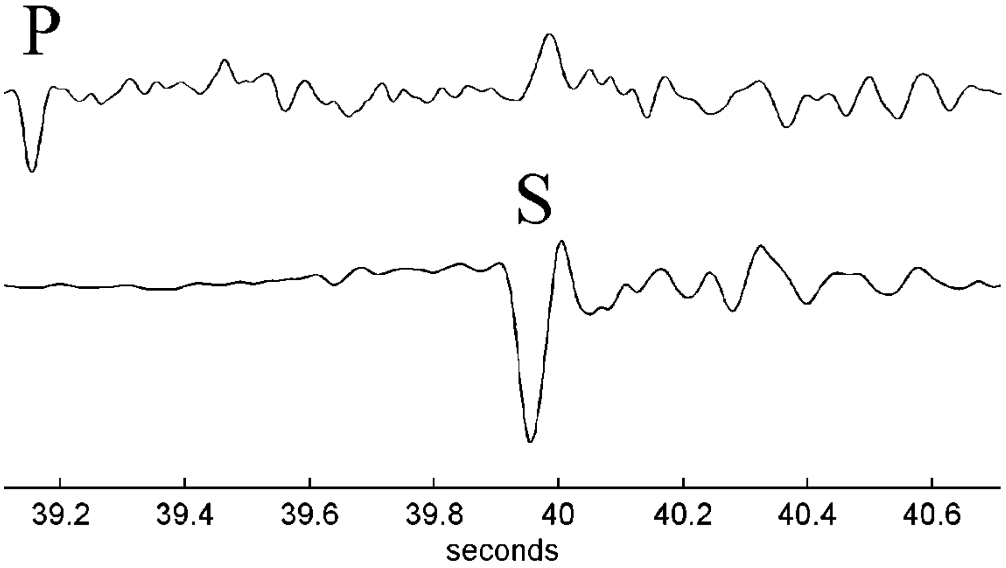
The far-field displacement pulse is equivalent to the apparent source-moment-rate function



Seismic moment: $M_0 \propto \int_0^T u^{SH}(\tau)d\tau = c_s AT$

(2) $\log A = \log M_0 - \log T - \log C$

Basel M_L 3.4 event recorded by borehole sensor OTER1 at 500 m.



Instrument corrected displacement rotated to max P- and S-amplitude.

Pulse duration: $T = L/v_a$

L = the source dimension (source radius for a circular fault)
v_a = apparent rupture velocity

(3) $$\log T = \log L - \log(v_a)$$

Seismic moment: $M_0 = \mu S \bar{D}$

Static stress drop: $\Delta\sigma_s = K\mu \frac{\bar{D}}{W}$

W and L fault width and length
$a_r = W/L$ aspect ratio
$W = a_r L$ and $S = a_r L^2$

$$\Delta\sigma_s = \frac{K}{a_r^2}\frac{M_0}{L^3}$$

(4) $$\log L = \frac{1}{3}\log M_0 - \frac{1}{3}\log(\Delta\sigma_s) + \frac{1}{3}\log K - \frac{2}{3}\log a_r$$

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(3) $$\log T = \log L - \log(v_a)$$

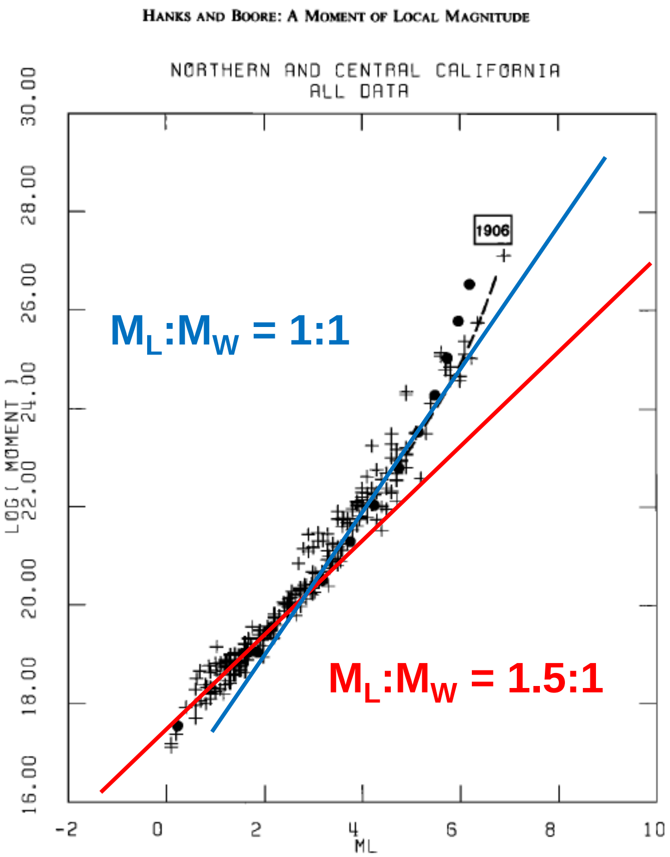
(2) $$\log A = \log M_0 - \log T - \log C$$

(1) $$M_L = \log A - \log A_0$$

(5) $$M_L = \frac{2}{3}\log M_0 + \boxed{\frac{1}{3}\log(\Delta\sigma_s) + \log(v_a) - \frac{1}{3}\log K + \frac{2}{3}\log a_r - \log C - \log A_0}$$

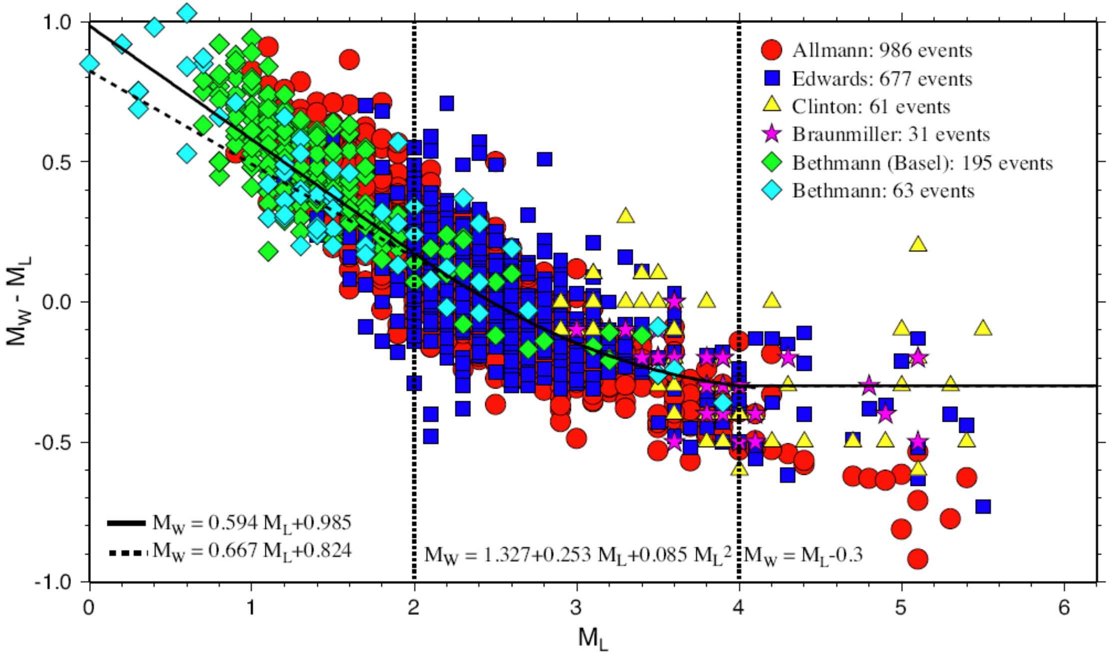
$$M_L = \frac{2}{3}\log M_0 + B \quad \Longleftrightarrow \quad M_W = \frac{2}{3}\log M_0 - 6$$

In a perfectly elastic medium and ignoring the Wood-Anderson response, ML and MW of self-similar earthquakes should scale 1:1



(Hanks & Boore, JGR, 1984)

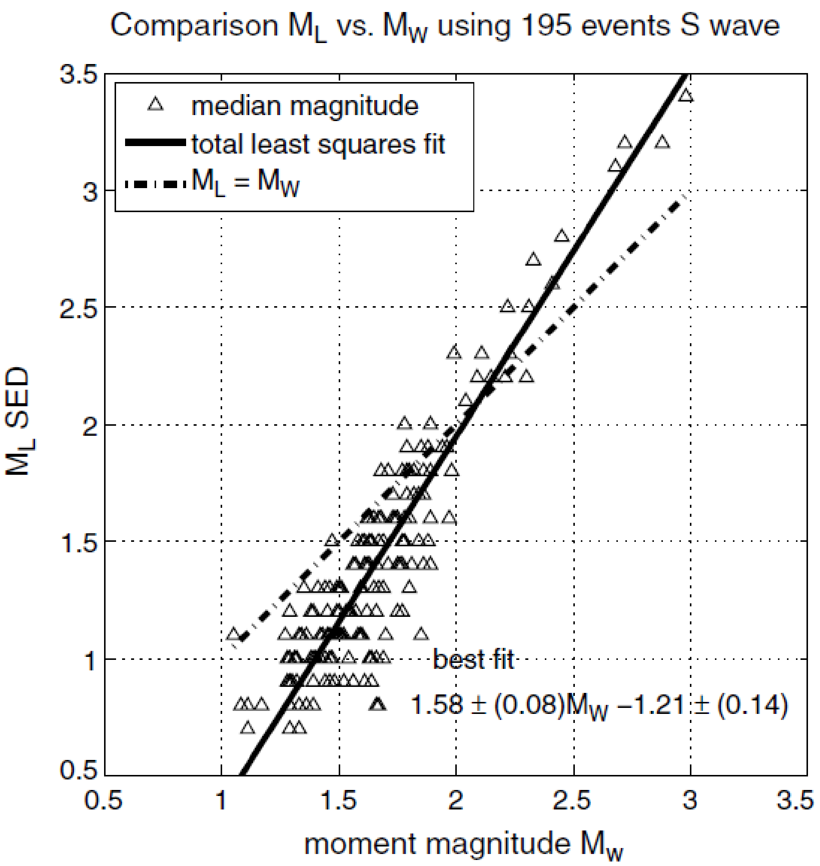
Switzerland



(Goertz-Allmann et al., BSSA, 2011)

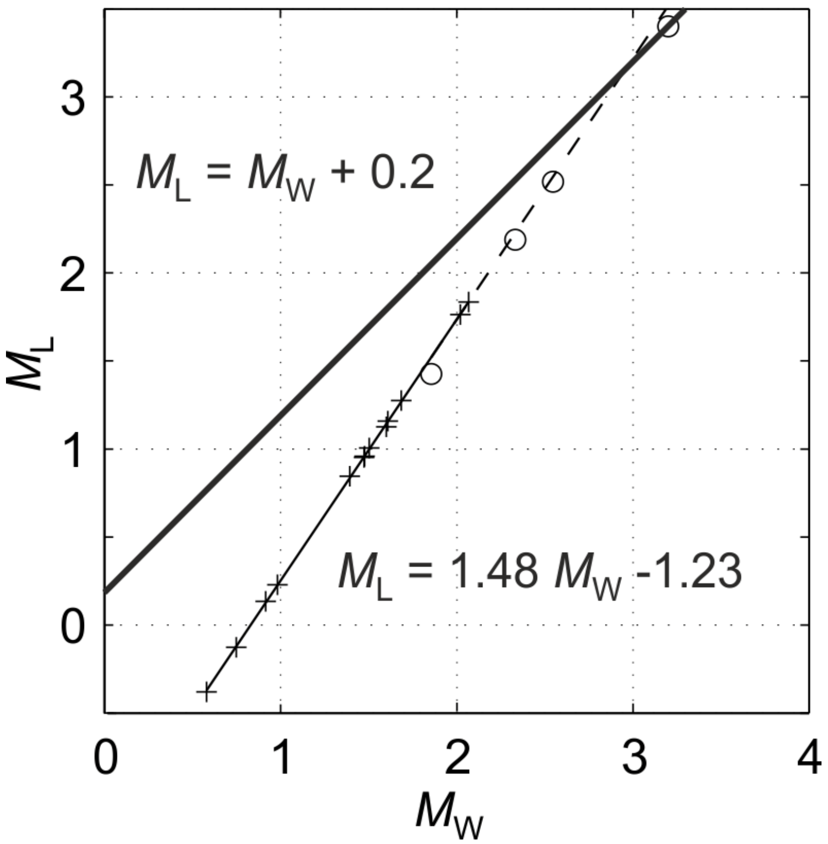
(Numerous other similar observations are documented in the literature)

Basel: induced seismicity 2006-2007



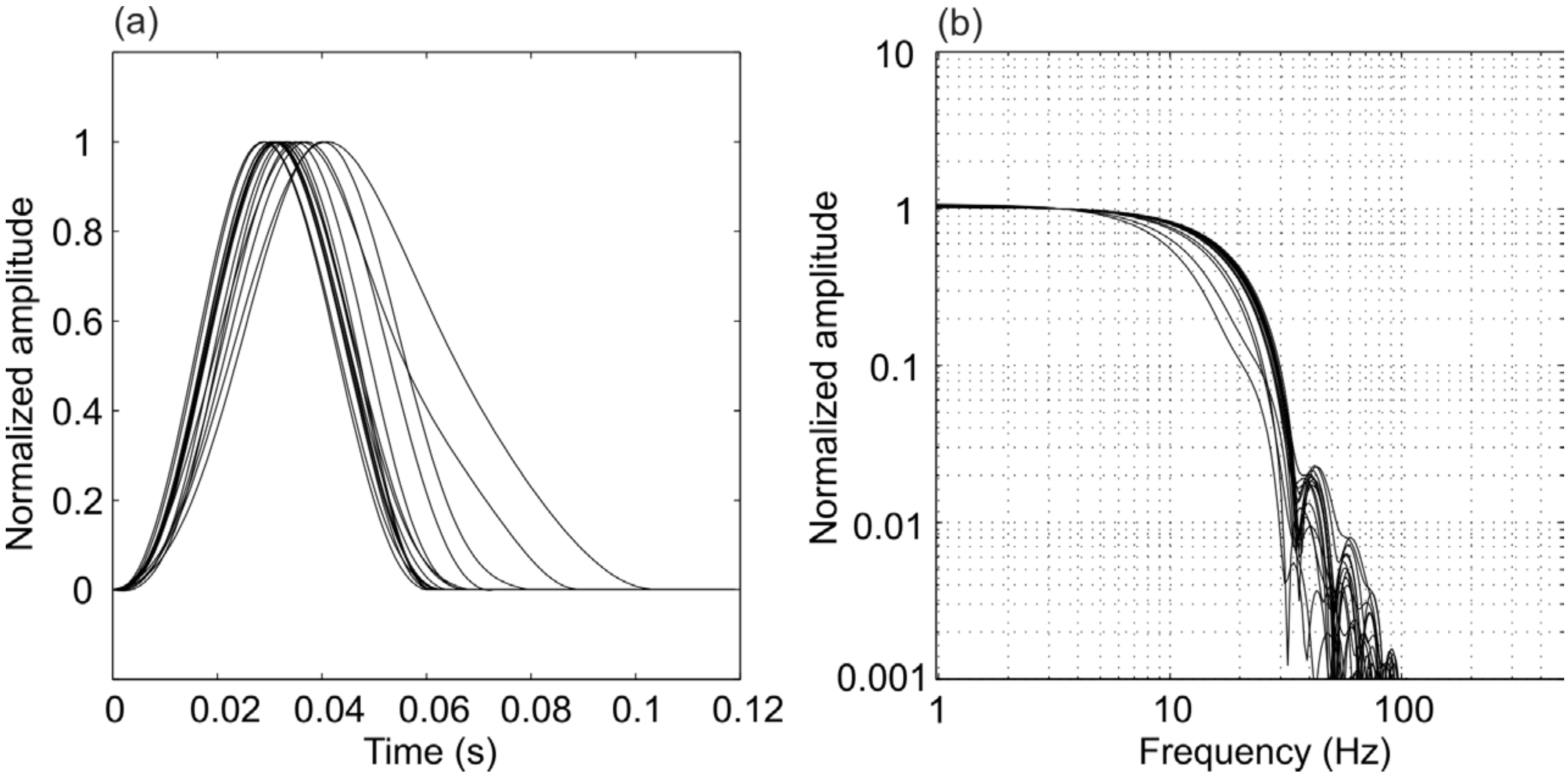
(Bethmann et al., BSSA, 2011)

Basel: cluster of closely co-located events with similar focal mech.



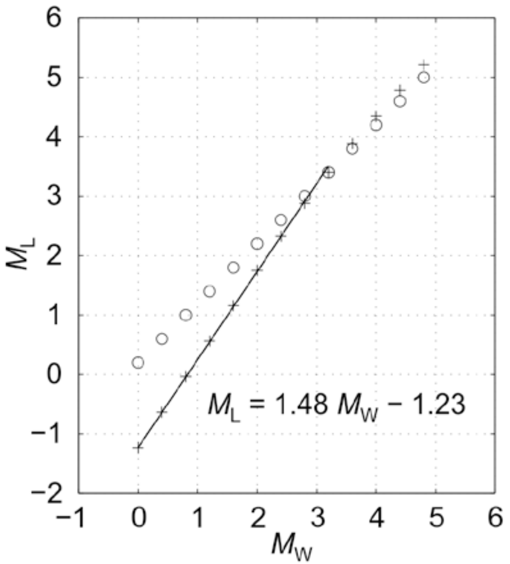
from a single borehole seismometer (MATTE at 553 m depth)

Pulse widths and displacement spectra of Basel cluster



Pulse widths and corner frequencies remain constant for $M_L < 2$

Synthetic moment-rate simulations of observations at borehole station MATTE in Basel

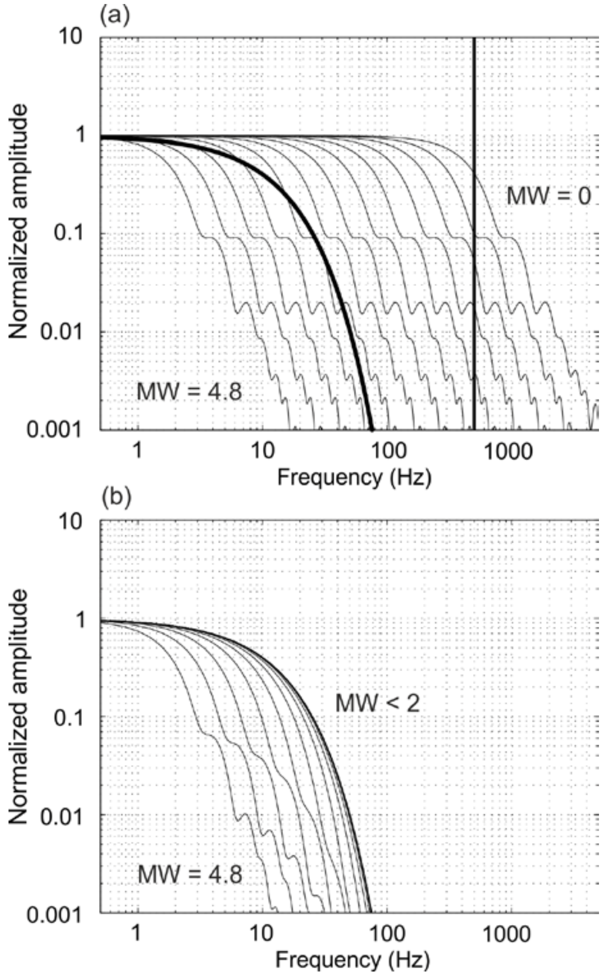


including attenuation

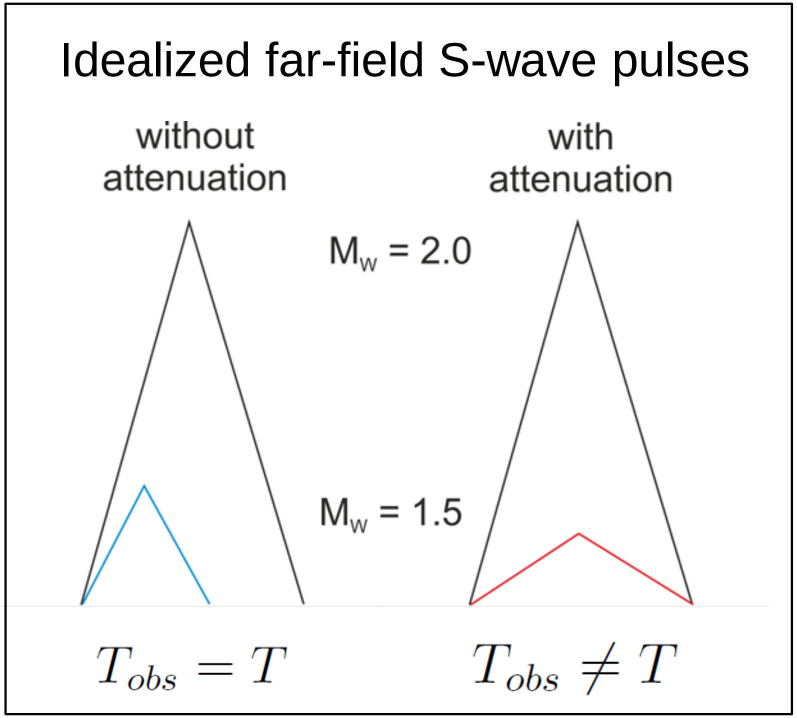
$$|A_Q(f)| = e^{-\pi t^* f}$$

$$t^* = \frac{x}{cQ}$$

Qs = 80, from spectral ratios
(Bethmann et al., GJI, 2012)



Circular source model with variable stress drop and rupture velocity (Deichmann, BSSA, 1997)



$$T = L/v_a$$
$$T_{obs} = T + kt^*$$
$$kt^* >> T$$
$$T_{obs} = T + kt^* \approx const.$$

Why 1.5:1 ?

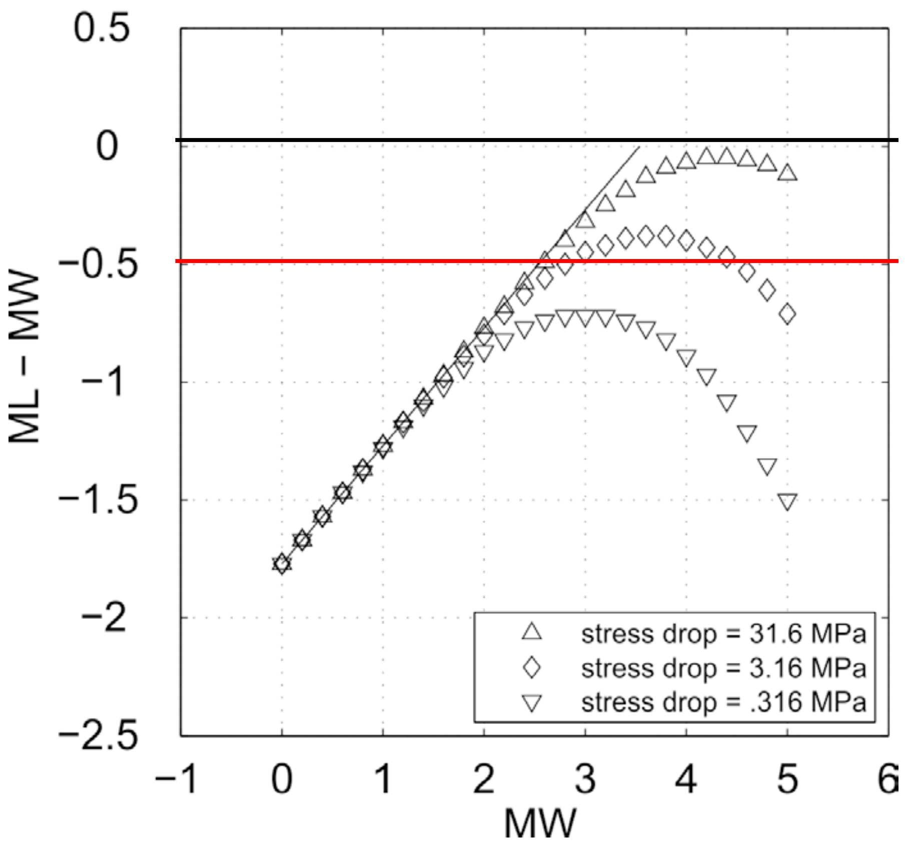
$$\log A = \log M_0 - \log T - \log C$$

$$\log A = \log M_0 + const.$$

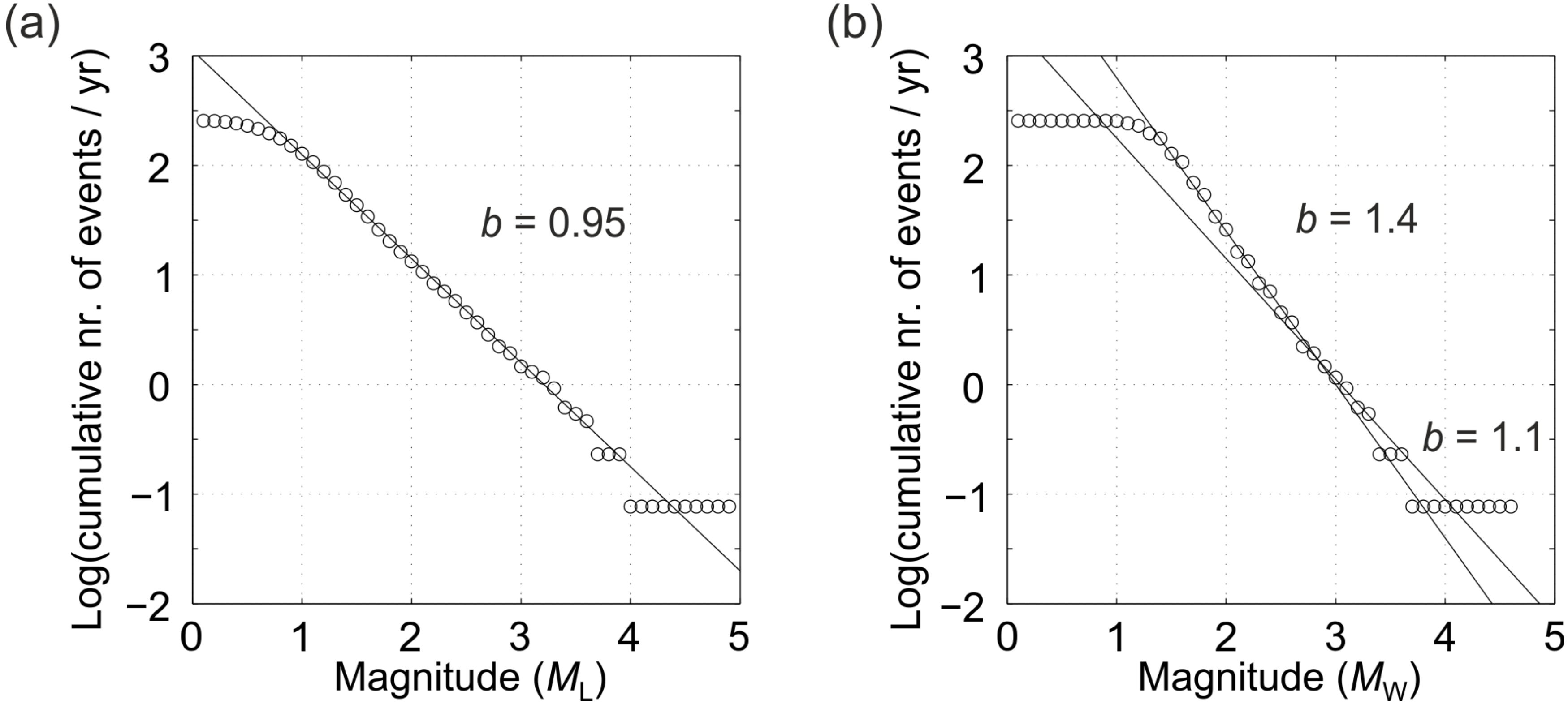
$$\log M_0 = \frac{3}{2} M_W + 9$$

$$M_L \propto \frac{3}{2} M_W$$

The effect of the Wood-Anderson response (with Q) for different stress drops



Consequences for G-R relations



3318 earthquakes from SW Switzerland (2002-2014)

Conclusions

Large earthquakes: $T_{obs} \propto M_0^{1/3} \quad (f_c \propto M_0^{-1/3}) \Longrightarrow M_L \propto M_W$

Small earthquakes: $T_{obs} = const. \quad (f_c = const.) \Longrightarrow M_L \propto \frac{3}{2} M_W$

- In practice, magnitudes of small and large earthquakes are like apples and pears
- The G-R relation based on M_L lacks physical justification
- The G-R relation based on M_W leads to different b-values for small and large earthquakes – we risk using the number of apples on an apple tree to estimate the number of pears on a pear tree

- For more insight into this we need
  - A Wood-Anderson-free magnitude
  - Native M_W values over a sequence of earthquakes with M_W from 0 to 6

(for details see Deichmann, BSSA, 107, 2, 2017)