How to Define the “Observed” Hazard When Validating Seismic Hazard Models?

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Abstract Previous studies that use observed data to validate earthquake hazard models, both the number of observation sites with at least one exceedance and the total number of exceedances at all available sites are used to represent the “observed hazard”. We compare the seismic hazard generated by the open software CRISIS for the Shanxi Rift System, NW-China with a seismic catalogue dating back to around 1500 A.D. with completeness magnitude 5.0 of the same area. We show that, “observed hazard “ represented by the total number of exceedances at all available sites is compatible with the hazard definition of Cornell’s PSHA method.

❖ Definition of Symbols:

Symbols	Meaning
T_R	return period, like 475-year and others
T_C	catalog time length, here is 567 year
$i = 1,2,\ldots,N_s$	number of sites
$n = 1,2\ldots,N_C$	number of mag≥5 earthquakes, here is 150
$k = 1,2,\ldots,N_k$	number of realizations
σ	standard deviation of GMPE function used
z_i	model predicted PGA at site i for T_R=475 year
$\varepsilon_{i,n,k}$	random variable, and $\varepsilon \sim N(0,\sigma^2)$
$g_{i,n}$	median PGA caused by historic EQs at site i

A: Cornell-type Method

If the logarithm of ground motion is normally distributed, the annual “observed hazard” at site i generated by N_C earthquakes within catalogue time T_C can be written as:

$$\lambda_{C,i} = \frac{1}{T_C} * \sum_{n=1}^{N_C} \Phi\left(\frac{g_{i,n}-z_i}{\sigma}\right)$$

Following the Poisson distribution assumption, the corresponding probability of exceedance at site i within 50-year is:

$$P_{C,i} = 1 - \exp(-\lambda_{C,i} * 50)$$

B: Stochastic Method

The stochastic method includes the ground motion uncertainty directly and generates many realizations $k = 1,2,\ldots,N_k$ of possible synthetic ground motions for the set of sources. For one realization k at location i, the ground motion generated by historical earthquake n is "$g_{i,n}+\varepsilon_{i,n,k}$". The random change of variable $\varepsilon_{i,n,k}$ follows normal distribution $N(0,\sigma^2)$. At each site, "$g_{i,n}+\varepsilon_{i,n,k}$" is directly compared with model predicted “"z_i” and we count in each realization how many earthquake generated ground motions enable $g_{i,n}+\varepsilon_{i,n,k} \geq z_i$. The mean exceedance rate at site i can be expressed as:

$$r_{k,i} = \sum_{n=1}^{N_C} H(g_{i,n} + \varepsilon_{i,n,k} - z_i)$$

H is the Heaviside function. For all the realizations N_k, the annual “observed hazard” at site i generated by N_C earthquakes within catalog time T_C can be formulized as:

$$\lambda_{S,i} = \frac{1}{T_C} * \frac{1}{N_k} \sum_{k=1}^{N_k} r_{k,i}$$

Thus, the corresponding probability of exceedance at site i within 50-year is:

$$P_{S,i} = 1 - \exp(-\lambda_{S,i} * 50)$$

❖ Implication

Using stochastic simulated ground motion we show that “observed hazard” should be calculated as the number of exceedances at a site averaged over all sites, rather than the average number of sites with at least once exceedance, in order to achieve consistency with the Cornell-type PSHA method.

In order to compare the ‘observed hazard’ with the ‘modelled hazard’ at the 475-year return period ground motion level we have to average over the ‘observed rates’.

❖ CRISIS Hazard Map

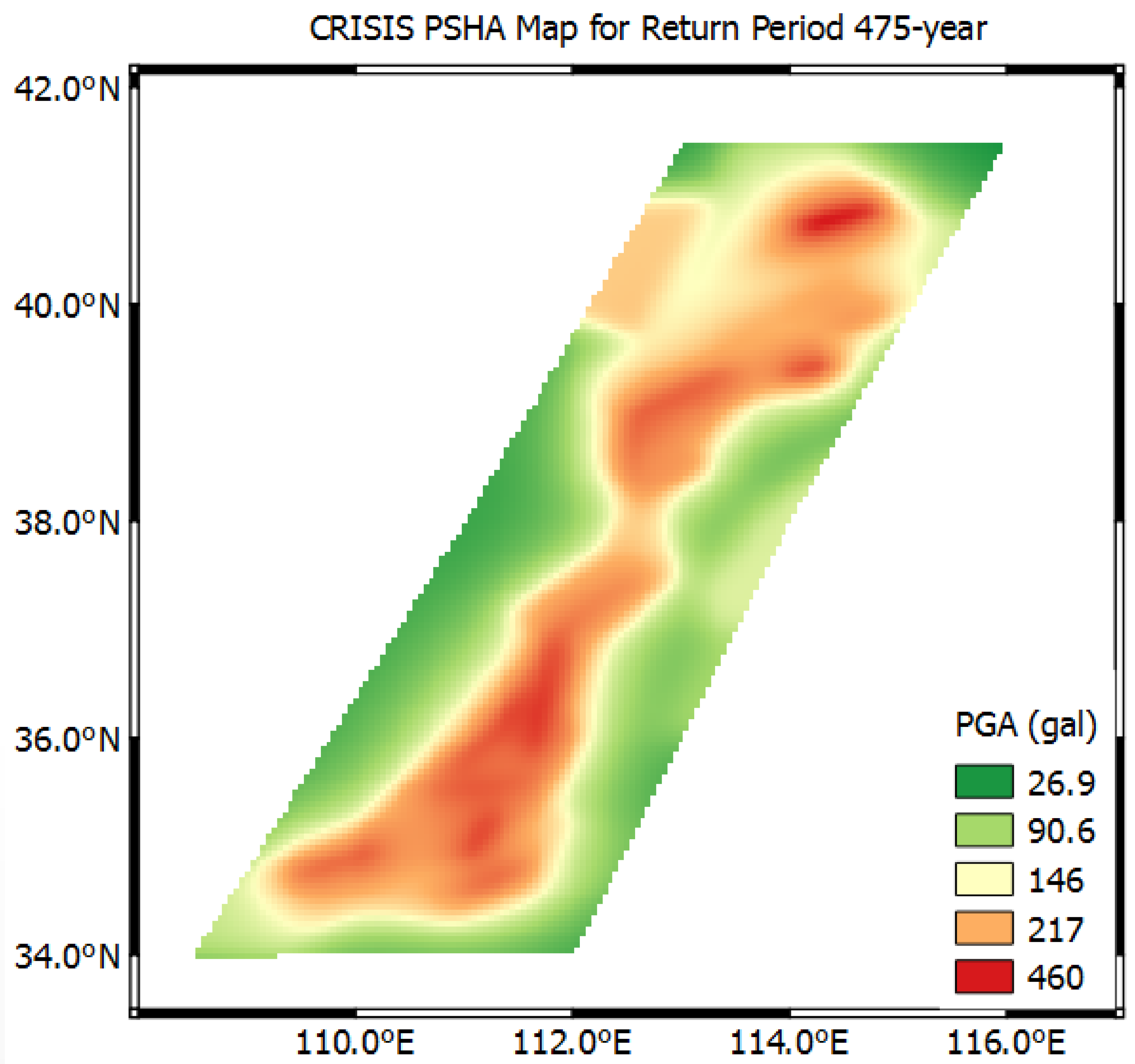

Fig. 1: CRISIS Hazard Map of 10% exceedance within 50-year.

❖ “Observed” Probability of Exceedance

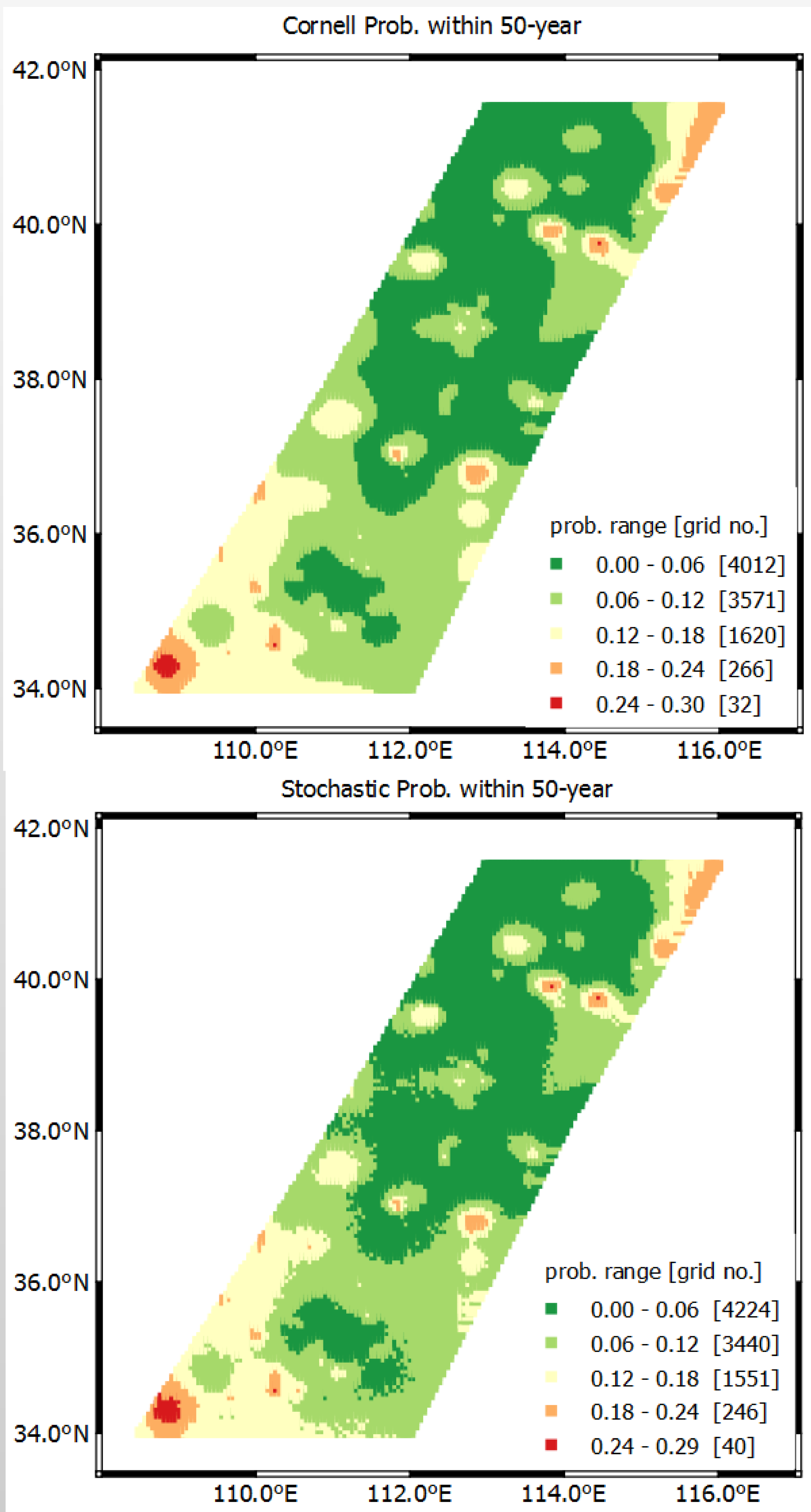

Fig. 2: The “observed” prob. of exceedance within 50-year at each grid calculated using Cornell (up) and stochastic method (down).

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